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An enhanced octree polyhedral scaled boundary finite element method and its applications in structure analysis

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ABSTRACT

In this paper, an enhanced octree polyhedral scaled boundary finite element method (SBFEM) is proposed in which arbitrary convex polygon (pentagon, hexagon, heptagon, octagon etc.) can be directly served as boundary face elements. The presented method overcomes the existing SBFEM's limitation that boundary face is strictly restricted to be a quadrangle or triangle. The conforming shape functions are constructed using a polygon mean-value interpolation scheme for polyhedral face. A highly efficient octree mesh generation technology is introduced to accelerate the progress of pre-treatment, wherein the mesh information can be directly used in the enhanced SBFEM. The accuracy of the proposed method is first verified using a beam under shear and torsion load. Another three more complicated geometries including a nuclear power plant structure, as well as two sculptures named Terra-Cotta Warriors and Sioux Falls Church are presented to demonstrate the application and robustness of the proposed method. The new method possesses appealing versatility and offers a swift adaptive capacity in mesh generation, which can provide a powerful technique for the simulation of complex geometries, rapid-design analysis and multi-scale problems.

1. Introduction

The finite element method (FEM) is a powerful computational technique used in numerical simulation that has been extensively applied to structural analysis since it was first proposed. Moreover, a wide variety of improvements and alterations to this method are continually emerging. The three-dimensional finite elements are typically tetrahedrons (four vertices and four faces) and hexahedrons (eight vertices and six faces). The hexahedron can degenerate into shapes with fewer vertices, such as pentahedrons (five or six vertices and five faces), which are sufficient for many applications. However, there is a growing need for more general polyhedral shapes with increasing geometric complexity to include shapes that have an arbitrary number of vertices and faces.

Polyhedral element shapes can provide more flexibility for meshing geometrically complex structures, which will enable a rapid design-to-analysis paradigm. Finite volume methods based on polyhedral cells have reached a level of maturity in fluid dynamic simulations, as evidenced by their availability and use in commercial software [1,2]. Mimetic finite difference (MFD) methods capable of handling general three-dimensional meshes have also been a topic of active research and have been successfully applied to diffusion, elasticity, and fluid flow problems [3–7]. However, the extension of the FEM to this field has been relatively slow despite the availability of special interpolation functions. This progress has primarily been slow because these interpolants are subject to restrictions on the admissible element geometry (e.g., convexity and maximum valence count) and can be sensitive to geometric degeneracies. More importantly, calculating these functions and their gradients are often prohibitively expensive. The numerical evaluation of weak form integrals with sufficient accuracy poses yet another challenge due to the non-polynomial nature of these functions as well as the arbitrary integration domain [8].

Polyhedral finite element formulations have only recently been proposed in the literature [9,10–13]. We highlight several works that have aimed to overcome these barriers. In his ground-breaking theoretical work that has drawn widespread attention from subsequent researchers, Wachspress [14] proposed polygonal rational shape functions for plane problems. However, this work was not extended to polyhedrons until 1996 [15] and was then extended to only polyhedrons with triangular faces. Similarly, Wicke et al. [11] developed a formulation for convex polyhedrons using mean-value coordinates where the faces were restricted to triangles. The polyhedral finite elements [10,13,16] were developed using the shape functions derived from meshless methods, and Idelsohn et al. [10] used non-Sibsonian coordinates that required a certain Voronoi construction within an element. To achieve numerical integration, the polyhedron was first subdivided into tetrahedrons, and

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tetrahedral subdivision was performed using the face centroids and the centroid of the polyhedron. Then, element integration was performed using standard integration rules for tetrahedrons [17]. Milbradt and Pick [13] developed a formulation for both convex and non-convex polyhedrons using natural-element coordinates to construct the shape function. Ghosh [18] developed the Voronoi-cell FEM using a stress-based finite element approach. In this formulation, the shape functions were constructed using a polynomial basis optimised for smoothness. More recently, researchers have focused on the virtual element method (VEM), which has addressed some of the aforementioned challenges encountered by finite element schemes [19–22].

The scaled boundary finite element method (SBFEM) was developed by Wolf and Song in the mid-1990s [23,24] and has been increasingly used for numerical simulations of structures. For example, Goswami [25], Hell [26], and Saputra et al. [27] used SBFEM to conduct three-dimension crack analysis. Man et al. [28] used this method to simulate plate bending, and Lin et al. [29] conducted a sloshing analysis of liquid storage tanks. An alternative hydrodynamic pressure method for a high concrete-faced rockfill dam has also been proposed [30]. Birk and Behnke [31] modelled dynamic soil-structure interactions in layered soil using a modified SBFEM. This method was also used to simulate electrostatic problems [32,33], short-crested wave interactions [34] and the time-domain analyses of the layered soil [35]. Novel consistent analysis approach for uniform beams [36], nonlinear analysis application in two-dimensional geotechnical structures [37] and error study of Westergaard's approximation using SBFEM [38] are emerged, too. Discretisation is conducted only at the boundary surface, and each face can be treated as a sub-element in each volume element, thus making the SBFEM easy to formulate for a polyhedral element. More recently, Liu et al. [39] developed a rapid automatic polyhedral mesh generation and scaled boundary finite element analysis. In this research, the octree mesh technique was adopted, and the two-dimensional isoparametric element was utilised for each face element. However, each face is still restricted to being either a triangle or quadrangle, and additional efforts are required to manage the hanging nodes, which may exceed the capacity of the method to manage general polyhedrons. This method will have greater influence if the hanging nodes are free from addressing.

In 1975, Wachspress [40] developed polygonal finite elements with an arbitrary number of sides. Subsequently, Meyer et al. [41] and Floater et al. [42] generalised these elements to arbitrarily shaped polygons using the concept of barycentric and mean-value coordinates. Voronoi diagrams and natural neighbour shape functions were considered to develop conforming polygon elements [43]. Recently, an alternative method using the maximum entropy approach [44] to formulate polygonal shape functions was also studied [45,46]. In summary, polygonal shape functions have reached a level of maturity that provides a powerful technique for formulating a general polyhedral element using SBFEM.

Based on previous studies, a more distinct three-dimensional polyhedral scaled boundary finite element method (PSBFEM3D) is proposed. The shape function of the boundary face elements is constructed using the mean-value coordinates instead of an isoparametric element. Subsequently, the solution process is identical to that of the SBFEM, which is described in detail in Section 3. The proposed method is easy to formulate and implement in a program. More importantly, the difficulties in calculating the shape functions and their gradients are significantly decreased compared with polyhedral FEM. In addition, this method can be seamlessly combined with efficient octree meshing, which is more advantageous for rapid adaptive modelling. The PSBFEM3D offers appealing universality and is efficient for managing a cross-scale or multi-scale mesh. This method may have significant potential for use in practical applications.

The remainder of this paper is organised as follows. A brief theoretical derivation of the mean-value shape function is introduced in Section 2. Section 3 describes the scaled boundary polyhedron formulation for a polyhedral element. The development platform of the proposed algorithm is introduced in Section 4. The reliability of the procedure is validated using four numerical examples in Section 5. Section 6 summarises the major conclusions that can be drawn from this study.

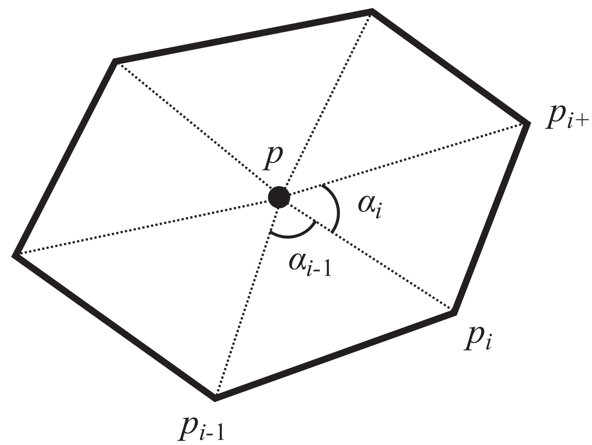

Fig. 1. Mean-value coordinates.

2. Mean-value shape functions on polygons

The mean-value shape function is a type of polygon function in which the interpolant is simpler and computationally attractive. The desirable properties for finite element interpolants indicated in reference [43] are satisfied, and the proof was presented in that previous study. Here, we briefly discuss the formulation of the mean-value shape functions on general polygon elements, and the reader can refer to [43] for a more detailed description. The linearly precise mean-value coordinate is written as [47]

$$N_i(\mathbf{x}) = \frac{w_i(\mathbf{x})}{\sum_{j=1}^{n} w_j(\mathbf{x})} \tag{2.1}$$

$$w_i(\mathbf{x}) = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{\|\mathbf{x} - \mathbf{x}_i\|} \tag{2.2}$$

$$\tan(\alpha_i/2) = \frac{\sin\alpha_i}{1 + \cos\alpha_i} \tag{2.3}$$

where $w_i(\mathbf{x})$ is the mean-value weight function, $||\mathbf{x}\text{-}\mathbf{x}_i||$ is the Euclidean distance between p and p_i (Fig. 1), point p is set as the geometric centre, and n is the number of vertices. The mean-value shape functions can be readily computed by substituting Eqs. (2.3) and (2.2) into (2.1). Notably, the formulation is also applicable to a non-convex polygon.

Typically, the mean-value shape functions are derived in the physical x-coordinate itself. To improve the SBFEM and to simplify the integral, a technique to construct conforming approximations on polygons is described using mean-value shape functions. Similar to isoparametric elements, the shape function is defined on a canonical element with local coordinates $\xi \equiv (\xi_1,\ \xi_2) \in \mathbf{R}_0$. The canonical elements are given for a triangle, quadrangle, pentagon and hexagon and are illustrated in Fig. 2. The nodes lie on the same circumcircle in each case, and the geometric centre is located in the centre of the circumcircle; hence, all vertices of the polygon are natural neighbours for any point in $\mathbf{R}_0$. The vertical coordinates of an n-gon are expressed as $(\cos 2\pi/n,\ \sin 2\pi/n)$, $(\cos 4\pi/n,\ \sin 4\pi/n)$,…, and $(1,\ 0)$. Then, the shape function can first be defined using the reference coordinate system. Subsequently, each arbitrary polygon can be transformed into corresponding canonical elements using the isoparametric mapping function $\boldsymbol{N}$. The mapping for a pentagonal element is illustrated in Fig. 3.