Engineering Analysis with Boundary Elements 84 (2017) 108–116

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Element-free Galerkin method for numerical simulation of sediment transport equations on regular and irregular distribution of nodes

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A R T I C L E I N F O

Keywords:
Meshfree method
Element-free Galerkin
Shallow water equations
Morphodynamic

A B S T R A C T

In this paper, the numerical simulation of the bed-load sediment transport using Element-Free Galerkin (EFG) method is presented. The governing equations of this model include shallow water equations for the hydraulic behaviour, Exner equation for morphodynamic variations, and Grass model for solid discharge. The governing equations are formulated for the coupled approach. The problem is solved using EFG meshfree method, in which the problem domain is represented by a set of arbitrarily distributed nodes; there is no need to use meshes, elements, or any other node connectivity information for field variable interpolation. The EFG method is based on moving least-squares (MLS) shape functions originated in scattered data fitting. Finally, to assess the ability and the efficiency of the EFG method, several benchmark examples on regular and irregular distribution of nodes are investigated and the results are compared with those of previously published works.

1. Introduction

Sediment transport in rivers has been studied for a long time with the aim to determine transporting estimation rates and corresponding changes in the topography of the river bed. The common approaches for computing sediment transport rates are mainly based on empirical relations obtained through the evaluation of field measurements or laboratory experiments. Because of the high cost of construction, maintenance, and operation, numerical modelling is being noticed by many researchers nowadays. Traditionally, some of the numerical methods like finite difference methods (FDM) and finite volume methods (FVM) were used to solve complex partial differential equations. These methods require the use of meshes or elements for spatial domain where the partial differential equations are governed. As mesh generation is a time-consuming process, it would be very profitable to introduce methods that do not demand prescribed node connectivity information or at least reduce the need for such information. As the numerical modelling field moved forward, other classes of solution methods were developed. Of particular note among these were meshfree methods.

The main idea of the meshfree method is to establish a system of algebraic equations for the whole problem domain and then approximate the field variable by means of a linear combination of shape functions that are built in the absence of predefined mesh generation. In these methods, problem domain is described only by dispersed nodes; no information about nodes connectivity is required and therefore the modelling of fracture, large deformation, etc. is considerably simplified [1]. A specific weight function is assigned to each of these nodes with local support domain. The shape function for a given node is then built by considering the weight functions whose support domain overlaps with the weight function of these nodes [2].

So far, different models have been proposed to determine the interaction between sediment transport and water flow. To quantify such interactions, it is necessary to develop numerical methods that accurately simulate flow and sediment transport. Generally, the mathematical modelling of morphodynamics is divided into two types of equations. The nonlinear shallow water equations describe the water flow continuity and momentum conservation with the assumption of shallow water. The Exner equation deals with sediment continuity to describe the bed updating. In the literature, different models of solid transport sediment flux have been proposed, such as Grass equation, Meyer–Peter & Muller's equation, Van Rijn's equation, and Nielsen's equation. These are generally acquired by empirical method [3–5]. In all of them except the Grass model, the critical shear stress controls the movement of sediment. In this paper, Grass model is considered for sediment transport flux.

Till date, different numerical methods have been used to solve the equations of sediment transport. A huge amount of work has been done using schemes based on FVMs. FVMs are obtained on the basis of the integral form of the conservative laws. They subdivide the spatial domain into grid cells and approximate the total integral of conservation laws over grid cells. The well-established Roe's scheme has been adapted to sediment transport problems (see e.g. [4,6–12]). The author of [9] mentions the comparison of implicit time advancing and explicit approach

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http://dx.doi.org/10.1016/j.enganabound.2017.08.007
Received 7 November 2016; Received in revised form 30 July 2017; Accepted 7 August 2017
Available online 31 August 2017
0955-7997/

in terms of accuracy and computational time and discretizes the governing equations using FVM and modified Roe's scheme in the second order of accuracy in space. Because of the treatment of source term in this scheme, it could increase the computational costs and is one of the disadvantages of this scheme. Some author have also used essential non-oscillatory (ENO) and weighted essential non-oscillatory (WENO) schemes to simulate the sediment transport problems [13], whilst others have employed the central weighted essential non-oscillatory (CWENO) scheme [14]. Most ENO, WENO, and CWENO schemes have simulated the two-dimensional sediment transport problems accurately, but unfortunately they are computationally still very expensive. The relaxation approach has also been applied [15]. In this approach, the nonlinear set of equations is transmuted to a semi-linear diagonalizable equation with linear characteristics variables. The second order MUSCL-TVD method is applied for the convection stage, whilst an implicit–explicit Runge–Kutta scheme solves the relaxation stage. The method of characteristics (MOC) is one of the best-known methods used in the solution and analysis of unsteady open channel flow problems [16]. The MOC, combined with the FVM and FVC methods, is used to solve the sediment transport problems [17]. Some research has been done using the finite element method [18,19]. Some authors have applied discontinuous Galerkin techniques to solve the sediment transport problems [18]. Space is discretized in this method using nodal polynomial basis functions of arbitrary order on each element of the unstructured computational domain. Also, in [19], the sediment transport equations were solved using the nodal discontinuous Galerkin finite element method. The unstructured computational domain is discretized by applying nodal polynomial basis functions of arbitrary order in space on each element.

All previously used numerical methods require some nodal connectivity information, known as mesh in finite element method, volume or cell in FVM, and grid in finite difference method. In the present work, a numerical model based on meshfree methods is applied for solving the sediment transport equations on regular and irregular distribution of nodes. The EFG method is a meshfree method advanced by Belytschko, it is based on MLS approximation [20,21]. The EFG method has been widely used to solve numerical problems (see e.g. [22,23]). For instance, the second order of elliptic partial equations was solved using this method and the convergent rate was investigated by using continuous and discontinuous shape functions [22]. This method only requires a set of nodes and a depiction of the boundaries to generate an approximation solution. The connectivity between the particular point of interest based on selected local nodes and the shape functions are constructed using the method without requiring elements.

This paper is organized as follows. In Section 2, the governing equations of sediment transport are formulated. In Section 3, the EFG method, which uses MLS shape functions to solve the sediment transport problem, is described. Thereafter, the numerical result of the test problem is presented in Section 4. In Section 5, the summarized concluding remarks are presented.

2. Governing equations

The shallow water equations depict the hydrodynamics part of the sediment transport model. The two space dimensions, the shallow water equations (neglecting the wind effects), Coriolis forces, and friction losses are presented as follows:

$$\begin{aligned} &\partial_t h + \partial_x(hu) + \partial_y(hv) = 0 \\ &\partial_t(hu) + \partial_x\left(hu^2 + \frac{1}{2}gh^2\right) + \partial_y(huv) = -gh\partial_x Z \\ &\partial_t(hv) + \partial_x(huv) + \partial_y\left(hv^2 + \frac{1}{2}gh^2\right) = -gh\partial_y Z \end{aligned} \tag{1}$$

Where t is the time, u and v are the depth-averaged velocities in x and y directions respectively, h is the height of water, g is the gravitational acceleration, and Z is the bottom topography. Note that Z is only the function of coordinates in shallow water equations, i.e. $Z = Z(x,y)$, and it is fixed in time. Therefore, when the sediment transport occurs, the bottom topography also depends on the time variable. Hence, an additional equation is needed for its evolution. Mathematically describing the continuity equation for sediment, the Exner equation is presented as follows:

$$\partial_t(Z) + \xi\left[\partial_x(Q_1) + \partial_y(Q_2)\right] = 0 \tag{2}$$

where $\xi = 1/1 - p$. The term $(1 - p)$ is sometimes called the packing factor and p is the sediment porosity which is assumed to be constant. In Eq. (2), Q_1 and Q_2 represent the bed-load sediment transport flux in x and y directions respectively. They are evaluated here using Grass model:

$$\begin{aligned} Q_1 &= A_g u\left(u^2 + v^2\right)^{\frac{m-1}{2}} \\ Q_2 &= A_g v\left(u^2 + v^2\right)^{\frac{m-1}{2}} \end{aligned} \tag{3}$$

Where $0 \le A_g \le 1$ and $1 \le m \le 4$ are experimental parameters that relate to the considered problem. A_g takes into account kinematic viscosity and grain diameter of the sediment and is also directly related to the strength of the interaction between the water flow and bed-load. If $A_g = 0$, sediments have no movement and we have a solid shallow water equation. When A_g is near zero, a weak interaction occurs between the water flow and sediment transport. When A_g approached 1, stronger interaction is presented. The system of equations can be rewritten in the coupled vector form as follows:

$$\begin{cases} \partial_t h + \partial_x(hu) + \partial_y(hv) = 0 \\ \partial_t(hu) + \partial_x(hu^2 + \frac{1}{2}gh^2) + \partial_y(huv) = -gh\partial_x Z \\ \partial_t(hv) + \partial_x(huv) + \partial_y(hv^2 + \frac{1}{2}gh^2) = -gh\partial_y Z \\ \partial_t(Z) + \xi\left[\partial_x(Q_1) + \partial_y(Q_2)\right] = 0 \end{cases} \tag{4}$$

The above system can be written as a non-conservative hyperbolic system:

$$\partial_t(W) + A_1\partial_x(W) + A_2\partial_y(W) = 0 \tag{5}$$

where

$$W = \begin{bmatrix} h \\ hu \\ hv \\ Z \end{bmatrix} \tag{6}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{{q_x}^2}{h^2} + gh & 2\frac{q_x}{h} & 0 & gh \\ -\frac{q_x q_y}{h^2} & \frac{q_y}{h} & \frac{q_x}{h} & 0 \\ -3A_g\xi\frac{q_x}{h^4}({q_x}^2 + {q_y}^2) & \frac{A_g\xi}{h^3}\left(3{q_x}^2 + {q_y}^2\right) & \frac{2A_g\xi}{h^3}\left(q_x q_y\right) & 0 \end{bmatrix} \tag{7}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{q_x q_y}{h^2} & \frac{q_y}{h} & \frac{q_x}{h} & 0 \\ -\frac{{q_y}^2}{h^2} + gh & 0 & 2\frac{q_y}{h} & gh \\ -3A_g\xi\frac{q_y}{h^4}({q_x}^2 + {q_y}^2) & \frac{2A_g\xi}{h^3}\left(q_x q_y\right) & \frac{A_g\xi}{h^3}\left({q_x}^2 + 3{q_y}^2\right) & 0 \end{bmatrix} \tag{8}$$

where W illustrates the vector of conserved variables, A_1 and A_2 are the coefficient matrices, $q_x = hu$ and $q_y = hv$ are the flow discharges in x and y directions, respectively.

3. Numerical formulation

In this work, the EFG method is used for simulation of the sediment transport equations. This section explains the MLS approximation shape functions considered for EFG method. After this, the EFG method is depicted for discretizing the governing equations.

3.1. MLS shape functions

In the MLS shape functions implemented most frequently in meshless methods, the unknown function ϕ is interpolated in $\mathbf{X}^T = [x, y]$ by the