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Combined mesh free method and mode matching approach for transmission loss predictions of expansion chamber silencers

Z. Fang[a,*], C.Y. Liu[b]

[a] School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, PR China
[b] China Ship Development and Design Center, Wuhan, Hubei 430064, PR China

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A B S T R A C T

A coupled method (abbreviated here as the MFMM method) that combines the mesh free (MF) method and mode matching (MM) approach is introduced to predict the transmission loss (TL) of expansion chamber silencers. The Hermite radial point interpolation collocation (HRPIC) method is employed to solve the 2-D eigen equation of silencer sections. Then the mode matching method is used to calculate the transmission loss. The eigenvalues predictions of circular cross section from the present method exhibit desirable precision comparison with analytical results. Additionally, the effects of size of influence domain, number of computational nodes and shape parameters of radial basis functions on the calculation accuracy are evaluated. TL calculations of four typical expansion chambers with extended inlet and outlet are presented to valid the computational accuracy and efficiency of the proposed combined method.

1. Introduction

The widely-used 3-D methods for predicting the acoustic attenuation performance of silencers are analytical mode matching (AMM) method and the numerical methods such as finite element method (FEM) and boundary element method (BEM). The AMM method [1–4] is confined to relatively simple configurations, such as circular, rectangular and elliptical silencers. The 3-D numerical methods (FE/BE method) [5–8] are able to cope with configuration complexities, but they are highly time consuming when the dimension of silencer is large or the computational frequency is high. Considering the restriction of the mode matching method and FE/BE method, the numerical mode matching (NMM) methods [9–13] which combine numerical methods with mode matching approach are developed to calculate the acoustic attenuation performance of silencers with arbitrary but axial uniform cross sections. The difference between AMM method and NMM method is the technique used in solving the eigen governing equation for the silencer section. AMM method uses the analytical approach to get the acoustic modes which are only suitable to the simple and regular sections. Acoustic modes of irregular shaped cross sections can only be achieved by using numerical methods, since the analytical solution is usually not available. The popular numerical methods such as the finite different method (FDM) [14], finite element method (FEM) [15] and boundary element method (BEM) [16] have been widely used to solve these problems. Mesh generation is an important stage in the pre-process of all the mesh-based methods, and it may be time-consuming, because that the problem domain must be properly meshed into elements of specific shapes, which is one of main bottlenecks of numerical simulations. In order to overcome these difficulties, the mesh free method (MFM) which no longer requires elements has been studied by many researches in recent decades.

Mesh free method [17] is a relatively new method in the field of numerical simulations. Field nodes independent of element are arbitrarily distributed inside the problem domain and on its boundary. Currently, there have been various popular mesh free methods such as the element free Galerkin (EFG) method [18], the reproducing kernel particle method (RKPM) [19], the mesh less local Petrov–Galerkin (MLPG) method [20], the boundary node method (BNM) [21], the point interpolation method (PIM) [22], the local point interpolation method (LPIM) [23], the point interpolation collocation method [24], the hybrid boundary node method (HBNM) [25], the boundary point interpolation method (BPIM) [26], the boundary knot method (BKM) [27], etc. In the last decades, many mesh free methods have been applied to investigate the eigen problems of acoustic cavity. Karageorghis [28] employed the method of fundamental solution for the eigenvalues calculation of the 2-D Helmholtz equation. Chen et al. [29] developed a boundary collocation method with RBF for solving the acoustic eigen frequencies of 3-D cavities. Christina and Otto [30] studied the dispersion effect of the Helmholtz equation in the 2-D case by using the radial point interpolation method (RPIM). The RPIM avoid the singular moment matrix comparing to the polynomial PIM. However, the numerical results are

* Corresponding author.
E-mail address: fangzhi@hust.edu.cn (Z. Fang).

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still instable when Neumann boundaries exist. Thus, Hermite RPIM technique is additionally proposed to deal with the Neumann boundary conditions. Investigation of combined Hermite RPIM with mode matching approach for acoustic transversal modes and attenuation performance of expansion chambers has not been conducted yet.

In this paper the Hermite RPIM with polynomial reproduction is used for shape function calculation, and the point collocation technique is used for solving the transversal eigen governing equation. Then the mode matching approach is applied to obtain the transmission loss of the expansion chamber silencers. The shape functions of filed nodes are derived by using the PRIM and Hermite RPIM in Section 2. Section 3 introduces the process of combination of mesh free method and mode matching approach. In Section 4 the effects of shape parameters on calculation error of eigenvalues are investigated and computational accuracy and efficiency of the proposed method are presented.

2. Theory

2.1. Acoustic governing equations

The expansions chamber is usually composed of some chambers and ducts with uniform cross-section. For the three-dimensional sound propagation in a chamber/duct with uniform cross-section, the sound pressure may be expressed as [31]

$$p(x,y,z) = p_{xy}(x,y)p_z(z) \tag{1}$$

where p is the sound pressure, p_{xy} and p_z represent the transversal and axial sound pressure components, respectively, and satisfy the following two independent equations:

$$\nabla^2_{xy}p_{xy} + k^2_{xy}p_{xy} = 0 \tag{2}$$

$$\frac{d^2p_z}{dz^2} + k_z^2 p_z = 0 \tag{3}$$

where ∇^2_{xy} is the Laplacian operator in the 2-D Cartesian coordinate system, k is the wavenumber, k_{xy} and k_z are the wavenumbers in the transversal and axial directions, respectively, and related with the compatibility condition

$$k^2_{xy} + k^2_z = k^2 \tag{4}$$

For the rigid wall, the transversal boundary condition may be expressed as

$$\partial p_{xy}/\partial n = 0 \tag{5}$$

For an arbitrary shaped cross-section, Eq. (2) may be solved numerically, such as the 2-D FEM, BEM or MFM, and theoretically there are infinite solutions. Therefore, the sound pressure may be written as

$$p(x,y,z) = \sum_{i=0}^{\infty} p_{xyi}(x,y)\left(C_1e^{-\mathrm{j}k_z z} + C_2 e^{\mathrm{j}k_z z}\right) \tag{6}$$

where i represents the order of transversal modes.

2.2. Hermite RPIM

In this paper the 2-D mesh free method is used to solve Eq. (2). Fields points are randomly distributed inside considered cross section domain Ω and its boundary Γ as shown in Fig. 1. For each computational point (x, y), an influence domain is defined, and an approximation p^h of the acoustic pressure is computed. In order to improve calculation accuracy, Hermite radial basis functions are employed to deal with the Neumann boundaries.

If the nodes on Neumann boundaries are included in the influence domain of (x, y), then its normal derivative is selected as additional unknown quantity. Thus, the approximation of the acoustic pressure p^h may be written as a liner combination of radial basis functions at

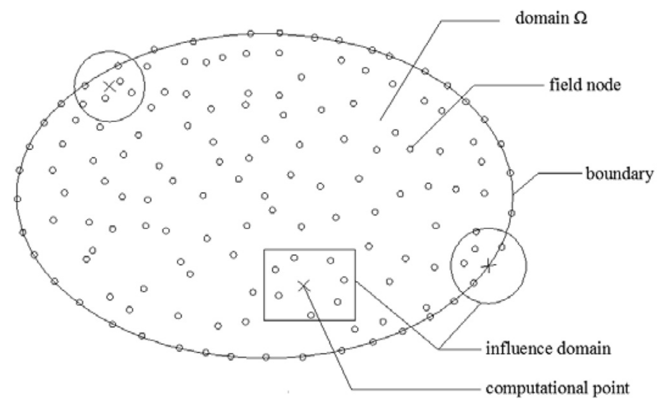

Fig. 1. The influence domain of computational point in mesh free method.

the n field nodes (contain nodes on Neumann boundaries) within influence domain, and its normal derivatives at the n_{DB} nodes on Neumann boundaries,

$$p^h = \sum_{i=1}^{n} R_i(\mathbf{x})a_i + \sum_{j=1}^{n_{DB}} \frac{\partial R_j^{DB}(\mathbf{x})}{\partial \mathbf{n}} b_j + \sum_{k=1}^{m} q_k(\mathbf{x})c_k \tag{7}$$

$$R_i(\mathbf{x}) = R(\|\mathbf{x}-\mathbf{x}_i\|) = \left(\left(\sqrt{(x_i-x)^2+(y_i-y)^2}\right)^2 + (\alpha_c d_c)^2\right)^q \tag{8}$$

$$R_j^{DB}(\mathbf{x}) = R\left(\left\|\mathbf{x}-\mathbf{x}_j^{DB}\right\|\right) = \left(\left(\sqrt{\left(x_j^{DB}-x\right)^2+\left(y_j^{DB}-y\right)^2}\right)^2 + (\alpha_c d_c)^2\right)^q \tag{9}$$

$$\frac{\partial R_j^{DB}(\mathbf{x})}{\partial \mathbf{n}} = l_{xj}\frac{\partial R_j^{DB}(\mathbf{x})}{\partial x} + l_{yj}\frac{\partial R_j^{DB}(\mathbf{x})}{\partial y} \tag{10}$$

Here, $R_i(\mathbf{x})$ is radial basis function and d_c is characteristic length. α_c and q are dimensionless shape parameters, and it is important to choose appropriate values to optimize the simulations. $\mathbf{x}_i$ are the coordinates of the nodes inside the influence domain, and a_i are corresponding coefficients. $\mathbf{x}_j^{DB}$ are the coordinates of the nodes on the Neumann boundaries, and b_j are the corresponding coefficients. q_k is the unknown polynomial, m is the items of the polynomials, and c_k are the coefficients of the polynomial. $\mathbf{n}$ is the unit normal vector at the jth point on Neumann boundaries, and l_{xj}, l_{yj} are the elements of the normal vector.

Eq. (7) can be rewritten as the following matrix formulations:

$$p^h(\mathbf{x}) = \mathbf{B}^{\mathrm{T}}\mathbf{a}_0 \tag{11}$$

$$\mathbf{B}^{\mathrm{T}} = \left[R_1 \cdots R_n \quad \frac{\partial R_1^{DB}}{\partial n} \cdots \frac{\partial R_{n_{DB}}^{DB}}{\partial n} \quad 1 \; x \; y \; \ldots \; q_m(\mathbf{x})\right] \tag{12}$$

$$\mathbf{a}_0 = \left[a_1 \cdots a_n \quad b_1 \cdots b_{n_{DB}} \quad c_1 \cdots c_m\right]^{\mathrm{T}} \tag{13}$$

In order to obtain the coefficients a_i, b_j and c_k in Eq. (7), we enforce Eq. (7) passing through all n nodes inside the influence domain and its normal derivatives' interpolations passing through all n_{DB} nodes on Neumann boundaries.

The interpolations of the function at lth point have the form,

$$p_l = p^h(\mathbf{x}_l) = \sum_{i=1}^{n} R_i(\mathbf{x}_l)a_i + \sum_{j=1}^{n_{DB}} \frac{\partial R_j^{DB}(\mathbf{x}_l)}{\partial \mathbf{n}} b_j + \sum_{k=1}^{m} q_k(\mathbf{x}_l)c_k, \quad l = 1,2,\ldots,n \tag{14}$$

The interpolations of the normal derivatives of function at the lth point on the Neumann boundaries can be expressed as

$$\frac{\partial p_l^{DB}}{\partial \mathbf{n}} = \sum_{i=1}^{n} \frac{\partial R_i(\mathbf{x}_l^{DB})}{\partial \mathbf{n}} a_i + \sum_{j=1}^{n_{DB}} \frac{\partial^2 R_j^{DB}(\mathbf{x}_l^{DB})}{\partial \mathbf{n}^2} b_j + \sum_{k=1}^{m} \frac{\partial q_k(\mathbf{x}_l^{DB})}{\partial \mathbf{n}} c_k, \quad l = 1,2,\ldots,n_{DB} \tag{15}$$