Engineering Analysis with Boundary Elements 84 (2017) 206–212

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A novel approach to solve inverse heat conduction problems: Coupling scaled boundary finite element method to a hybrid optimization algorithm

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A R T I C L E I N F O

Keywords:
Scaled boundary finite element method (SBFEM)
Inverse heat conduction
Hybrid optimization
GA-SQP

A B S T R A C T

Scaled boundary finite element method (SBFEM) has proved its abilities in problems with singularities successfully. In this work, the coupling of SBFEM and a hybrid optimization algorithm is employed to determine unknown heat flux in the transient heat conduction problems. The genetic algorithm (GA) is a stochastic method which solves problems considering a large number of generations, while the deterministic methods such as sequential quadratic programming (SQP), which are sensitive to the initial points, can solve problems faster. Combining GA, as the main optimizer, and SQP can lead to lower computational time. Herein, a square plate is considered as the case study. The inverse analysis is accomplished by utilizing the transient temperature data from direct solution. The difference between the calculated and the known values of temperature at four points within the plate is considered as the objective function, and the heat fluxes on the upper side of the plate are considered as the design variables. As a result, the exact value of the heat fluxes is obtained using this method. This new approach, in which the SBFEM as a meshless solver is combined with the hybrid GA-SQP as the optimizer, highlights its potentials in solving inverse problems.

1. Introduction

A traditional heat transfer analysis is accomplished by introducing material properties, initial condition and boundary conditions, which results in temperature distribution in the body. This type of heat transfer problem is identified as the direct heat transfer problem (DHTP). An enormous amount of work has been dedicated to the DHTP, and different method has been developed. Due to the small size of the body or limitation on operation range of probes, it may be a difficult matter to measure temperature or heat flux on the surface of a body experimentally. According to further information at some points of the body, these unknown boundary values can be estimated using an appropriate inverse method. In fact, the inverse heat transfer problems (IHTP) establish a close collaboration between the experimental and theoretical problems [1–4].

Recently, genetic algorithm (GA) has been the center of attention in the IHT problems, because of its privilege characteristics which are less dependent on the initial value and do not need any gradient information. The utilization of GA in heat transfer problems has been reviewed in Ref. [5]. The IHTP can be viewed as an optimization problem which is aimed to minimize the difference between the measured and the estimated values. Das et al. [6] combined the genetic algorithm, finite volume (FVM) and lattice Boltzmann (LBM) methods to predict unknown parameters. In their study, the FVM and LBM were used for obtaining radiative information and solving energy equation, respectively. The best values of unknown parameters were obtained using GA. Das [7] approximated the unknown heat transfer coefficient for a cylindrical fin, using GA. He also investigated the effect of some temperature measurements on the prediction of unknown parameter and concluded that a reasonable prediction was observed considering more than 50 temperature points on the body. He et al. [8] investigated a steady-state inverse heat transfer problem and employed multi-objective optimization to estimate the thermos-physical properties. They concluded that using the multi-objective genetic algorithm resulted in more accurate estimation which was capable of reducing the effects of noise and errors of measurements. Kim and Baek [9] compared different inverse methods including the conjugate-gradient method, the hybrid genetic algorithm and the finite difference Newton method for inverse radiation problem. They made a conclusion that the efficient results were obtained when

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http://dx.doi.org/10.1016/j.enganabound.2017.08.018
Received 8 April 2017; Received in revised form 29 July 2017; Accepted 28 August 2017
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the hybrid genetic algorithm was used as an initial value selector and the finite difference Newton method was applied as an estimator.

No matter what algorithm is chosen for IHTP, it requires an accurate method for a direct heat transfer calculation. The numerical solution of IHTP was started by Stolz [10] and since then, the diverse methods have been developed for these kinds of heat transfer problem. These methods can be categorized in mesh-based and meshless ways [11]. The traditional mesh-based methods are those which work with discretized mesh systems. The finite element [12,13], finite volume [14,15] and finite difference [16,17] methods are the most well-known mesh-based methods which work by generating mesh within the whole domain of the body. Reddy and Balaji [18] estimated a temperature dependent heat transfer coefficient for a rectangular fin, using the finite difference method in a way that led to minimizing the difference between Thermochromic Liquid Crystal (TLC) measurement and simulated temperature. The needs for solving complex engineering problems have made the scholars to develop an efficient and accurate numerical method which is cost-effective, quick and reliable in many cases. In the finite element method (FEM), a spatial discretization of the domain is carried out, and for each element, the temperature is interpolated using the shape function. The noticeable feature of FEM is its flexibility for mesh generation. The traditional FEM relies on the type of mesh employed for the problem. So, a balance is required between the accuracy and the flexibility in the mesh generation [19].

In the boundary element method (BEM), the boundary of the domain is discretized and reduces the dimensions of the problem by one. Furthermore, a basic solution, which satisfies the governing equation, is required. Sometimes, this analytical solution may lead to a complex problem. The scaled boundary finite element method (SBFEM), developed by Wolf and Song [20,21], benefits the advantages of both boundary element and finite element methods. In this method, the boundary is discretized with elements based on FEM, and numerical solution is done on the boundary. Then, the partial differential equation of the problem is transformed into an ordinary differential equation which can be solved analytically [20]. This method not only has a low computational cost rather than FEM, but also it does not need any fundamental solution, unlike BEM. The traditional boundary element methods have a strong limitation on a nonlinear problem that leads to divergence in iterative solution [22]. The SBFEM has been extended to the nonlinear problem [23]. Because of the aforementioned superiorities of the SBFEM in some engineering problems, a great tendency has been grown to employ this method as a solver. In spite of all advantages of SBFEM, the need for satisfying scaling center requirement causes a limitation in a complex geometry. But this matter is settled down by dividing the whole domain into smaller subdomains. Moreover, these subdomains can be coupled with the finite element [24] and boundary element methods [25].

The SBFEM has been applied in different engineering problems for example see Refs. [26–33]. He et al. [34] solved a two-dimensional steady state heat transfer problem using SBFEM. In their study, to reduce the computational cost, a cyclically symmetry was used and proved that the matrix coefficients in SBFEM were block-circulant in a cyclically symmetric system. Bazyar and Talebi [35] represented the advantages of SBFEM by solving a transient heat transfer problem for different complex geometries. Lin and Liao [23] combined the traditional SBFEM with the homotopy analysis method to make this method more beneficial for nonlinear problems and then applied the modified SBFEM for a nonlinear heat transfer problem.

This study is focused on the investigation of inverse heat transfer problem using a new method as a thermal solver. Herein, the scaled boundary finite element method is selected as the heat transfer solver, and the hybrid algorithm (GA-SQP) is employed to determine the unknown boundary conditions in a transient IHT problem. The SBFEM benefits the advantages of both finite element and boundary element methods, and has made it an applicable solution method for engineering problems including heat transfer. Also, such an extension of SBFEM to solve an inverse heat transfer problem has not been reported yet.

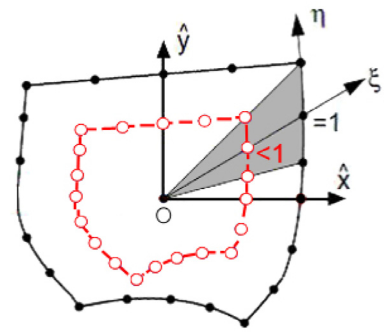

Fig. 1. Spatial discretization of scale boundary finite element method.

2. Mathematical model

2.1. Scaled boundary finite element method (SBFEM)

The derivation and solution procedure of the scaled boundary finite element method are given in [20,21]. In this section, a brief review and key concepts of this method are presented. For illustrating the concept of SBFEM, a two-dimensional bounded domain is considered as shown in Fig. 1.

The location of scaling center should be selected in a way that the entire boundary is visible from this point. Then, the boundary of the medium is divided into elements. The coordinates of an element on the boundary are defined by {x} and {y}. The interpolation of the element is done using a shape function which is denoted by [N(η)]. The points in the domain are obtained by scaling the boundary with the dimensionless coordinate ξ which varies from $\xi = 1$ on the boundary to $\xi = 0$ on the scaling center. The coordinates (ξ, η) are scaled boundary coordinates. So, ($\hat{x}, \hat{y}$) is used to define the geometry of interior points:

$$\hat{x}(\xi, \eta) = \xi\left[N(\eta)\right]\{x\} \tag{1}$$

$$\hat{y}(\xi, \eta) = \xi\left[N(\eta)\right]\{y\} \tag{2}$$

In other words, Eqs. (1) and (2) are the transformation from Cartesian coordinates ($\hat{x}, \hat{y}$) to scaled boundary coordinates (ξ, η).

For the purpose of transforming the heat transfer equation into the scaled boundary coordinates, it is needed to transform the spatial derivatives as:

$$\begin{Bmatrix} \partial/\partial\xi \\ \partial/\partial\eta \end{Bmatrix} = \left[\hat{J}(\xi, \eta)\right]\begin{Bmatrix} \partial/\partial\hat{x} \\ \partial/\partial\hat{y} \end{Bmatrix} = \left[b^1\right]\frac{\partial}{\partial\xi} + \frac{1}{\xi}\left[b^2\right]\frac{\partial}{\partial\eta} \tag{3}$$

where

$$\left[b^1\right] = \frac{1}{\left|\hat{J}\right|}\begin{Bmatrix} \left[N(\eta)\right]_{,\eta}\{y\} \\ -\left[N(\eta)\right]_{,\eta}\{x\} \end{Bmatrix} \tag{4}$$

$$\left[b^2\right] = \frac{1}{\left|\hat{J}\right|}\begin{Bmatrix} -\left[N(\eta)\right]\{y\} \\ \left[N(\eta)\right]\{x\} \end{Bmatrix} \tag{5}$$

and $\hat{J}(\xi, \eta)$ is the Jacobian matrix which contains partial derivative of $\hat{x}, \hat{y}$ on the ξ, η , and is defined as:

$$\left[\hat{J}(\xi, \eta)\right] = \begin{bmatrix} \hat{x}_{,\xi} & \hat{y}_{,\xi} \\ \hat{x}_{,\eta} & \hat{y}_{,\eta} \end{bmatrix} \tag{6}$$

The temperature of an element is interpolated using the shape function as following:

$$\{T(\xi, \eta)\} = \left[N(\eta)\right]\{T(\xi)\} \tag{7}$$

{T(ξ)} is introduced as nodal temperature function along radial lines from scaling center to the boundary.

After transformation of the heat transfer equation from the Cartesian coordinates to the scaled boundary coordinates, employing the