Engineering Analysis with Boundary Elements 84 (2017) 231–236

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A mesh-free vibration analysis of strain gradient nano-beams

Lifeng Wang [a,b,*], Xiaoqiao He [b], Yuzhou Sun [b,c], K.M. Liew [b,*]

[a] State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[b] Department of Architecture and Civil Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China
[c] School of Civil Engineering and Architecture, Zhongyuan University of Technology, Zhengzhou 450007, China

ARTICLE INFO

Keywords:
Mesh-free method
Strain gradient
Euler beam
Single-walled carbon nanotube
Vibration

ABSTRACT

This paper focuses on developing a mesh-free method to analyze vibrational behavior of strain gradient nano-beams. For this purpose, the paper starts with the dynamic equation of a strain gradient Euler beam, and then the moving least-square (MLS) approximation is used to construct the shape function and its second- and third-order derivatives. A mesh-free numerical simulation scheme is develop, in which the higher-order gradient of strain is directly approximated with the nodal components due to the higher-order continuity of the shape function. The reliability of the mesh-free method is illustrated by an example of the simply-supported beam. Numerical simulations are carried out to study the small scale effect on both natural frequencies and vibration mode shapes of a single-walled carbon nanotube (SWCNT) which can be modeled as a nano-beam. The results of the mesh-free analysis are in good agreement with the theoretical results in analyzing the simply-supported SWCNT. The difference of natural frequency between that predicted by the strain gradient elastic beam and the classical beam rises with the increasing of the mode order and decreasing of the length.

1. Introduction

In recent years, numerous researches demonstrated the appearance of scale effects due to microstructure at the micro and nano-scale [1–3]. The scale effect can be successfully modeled by employing the nonlocal elastic theory, in which the constitutive equations include some material parameters of length scale, in addition to the classical material parameters. The general nonlocal theory includes the theory with higher-order stress gradient by Eringen [4], the couple stress theory and strain gradient elasticity theory [5]. Toupin [6] and Mindlin [7] formulated the theory of strain gradient elasticity in 1960s. Aifantis [8,9] suggested a simple strain gradient theory. Different from the conventional elasticity, the stress of structure made of strain gradient material considers not only the strain of this point, but also the higher-order gradient of strain.

The beam model is very efficient for studying dynamic behaviors of slender nanostructures such as carbon nanotubes (CNTs) or nanowires. The study of vibration behavior for CNTs is of particular interest for their potential applications [10,11]. Since controlled experiments remain difficult and computationally expensive for atomistic simulation, elastic continuum models have been widely applied to CNTs in vibration analysis [12–14]. Recently, several research teams have implemented nonlocal continuum models to describe the size effect of nanostructure [15–17]. Most of these studies are mainly on the theoretical analysis of very simple support boundary conditions, but numerical simulations are rare due to the complexity induced by the higher-order gradient of strain. Only a few works on the finite element method or mesh-free method have been developed for complex boundary conditions and structures cases [18–22]. But, solving of the boundary value problems is complicated with the inclusion of the higher-order gradient of strain. The interpolation requires C^1-continuity in the finite element method, leading to difficulty in the establishment of elements and the construction of interpolation functions [23]. The mesh-free method has attracted more and more attention from researchers in recent years, and it is regarded as a potential numerical method in computational mechanics. After Belytschko et al. [24] put forward the element free Galerkin method, various mesh-free methods such as reproducing kernel particle method (RKPM) [25], improved moving-least squares Ritz method (IMLS-Ritz) [26], boundary node method (BNM) [27], point interpolation method (PIM) [28–32], have been developed and applied to static and dynamic problems of wide field. Recently, mesh-free methods have been applied to simulate the materials with strain gradient effects [33–38]. The mesh-free method has some distinct advantages. It does not require a mesh to discretize the problem domain, and the approximate solution is constructed entirely based on a set of scattered nodes, so the mesh-free shape function has better continuity and smoothness. In particular, the moving least-square (MLS) approximations possess nonlocal properties and satisfy the higher-order continuity requirement

* Corresponding authors.
E-mail addresses: walfe@nuaa.edu.cn, walfe@sohu.com (L. Wang), kmliew@cityu.edu.hk (K.M. Liew).

http://dx.doi.org/10.1016/j.enganabound.2017.09.001
Received 17 July 2016; Received in revised form 21 August 2017; Accepted 5 September 2017
Available online 20 September 2017
0955-7997/

automatically. This intrinsic nonlocal property leads to real rotation free approximation [33], thus displacements can be used as the only nodal freedoms. To the authors' best knowledge, however, only a few works have been available for the use of mesh-free model in dealing with vibration of strain gradient nano-beams with small scale effects taken into consideration.

The present work attempts to employ the mesh-free method to study the vibration behavior of strain gradient beams with size effect. The paper begins with a strain gradient Euler beam model. Next, a mesh-free beam model based on the MLS approximations for analysis the vibrational behavior of the nano-beam is presented based on the principle of virtual work. Then numerical simulations of mesh-free method are carried out to study the small scale effect on both natural frequencies and vibration modes of a SWCNT which can be modeled as a nano-beam. Finally, some concluding remarks are drawn.

2. Strain gradient Euler beam model

The nonlocal elasticity which considers a lattice structure is rising again to model the nanostructures. The constitutive law characterizing the nonlocal elastic material in a one-dimensional case reads [3,39]

$$\sigma_x = E\left(\varepsilon_x + r^2\frac{\partial^2\varepsilon_x}{\partial x^2}\right), \tag{1}$$

where E represents Young's modulus, ε_x the axial strain and σ_x axial stress, r is a material parameter to reflect the influence of the microstructure in the nonlocal elastic material. Then the dynamics equation of strain gradient Euler beam can be expressed as

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\left(\frac{\partial^4 w}{\partial x^4} + r^2\frac{\partial^6 w}{\partial x^6}\right) = 0. \tag{2}$$

For a simply-supported beam, the boundary conditions can be simply assumed as

$$\begin{aligned} &w(0) = 0, \quad w''(0) = 0, \quad w^{(4)}(0) = 0, \\ &w(L) = 0, \quad w''(L) = 0, \quad w^{(4)}(L) = 0, \end{aligned} \tag{3}$$

where L is the length of the beam. We assume the deflection of the simply-supported beam as

$$w(x,t) = \hat{w}\sin\frac{k\pi}{L}e^{i\omega t}, k = 1,2,3,\cdots, \tag{4}$$

where $\hat{w}$ is a constant, ω is angular frequency and $i \equiv \sqrt{-1}$. Substituting Eq. (4) into Eq. (2), we have the angular frequency of strain gradient vibration for the simply-supported beam

$$\omega = \left(\frac{k\pi}{L}\right)^2\sqrt{\frac{EI}{\rho A}\left[1 - r^2\left(\frac{k\pi}{L}\right)^2\right]}. \tag{5}$$

If $r = 0$, Eq. (5) leads to

$$\bar{\omega} = \left(\frac{k\pi}{L}\right)^2\sqrt{\frac{EI}{\rho A}}. \tag{6}$$

This is the angular natural frequency of a classical simply-supported Euler beam. The frequency ratio between natural frequency given by the strain gradient elastic beam and classical beam can be defined as

$$\frac{\omega}{\bar{\omega}} = \sqrt{1 - r^2\left(\frac{k\pi}{L}\right)^2}. \tag{7}$$

3. MLS mesh-free model for vibration of strain gradient beam

Considering the higher-order derivative of nonlocal elastic theory compares to the conventional elasticity theory, the function of strain is necessarily C^1-continuous in a discretizing procedure. The MLS approximation is used to build the numerical discretization scheme. The virtual work principle is used to obtain a mesh-free model for vibration of the strain gradient beam. The virtual work for vibration of one-dimensional case can be expressed as

$$\iiint_V E\left(\varepsilon_x + r^2\frac{\partial^2\varepsilon_x}{\partial x^2}\right)\delta\varepsilon_x dV + \iiint_V \rho\frac{\partial^2 w}{\partial t^2}\delta w dV = 0. \tag{8}$$

Applying the Green divergence theorem, one can obtain the virtual work function for vibration of a beam as

$$\begin{aligned} &\int_\Omega EI\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2(\delta w)}{\partial x^2} - r^2\frac{\partial^3 w}{\partial x^3}\frac{\partial^3(\delta w)}{\partial x^3}\right)dx \\ &+ \int_\Omega \rho A\frac{\partial^2 w}{\partial t^2}\delta w dx + EIr^2\frac{\partial^3 w}{\partial x^3}\frac{\partial^2(\delta w)}{\partial x^2}\bigg|_\Gamma = 0, \end{aligned} \tag{9}$$

where Γ is the boundary of the domain Ω. The boundary condition is set as

$$EIr^2\frac{\partial^3 w}{\partial x^3}\frac{\partial^2(\delta w)}{\partial x^2}\bigg|_\Gamma = 0. \tag{10}$$

So, Eq. (9) can be rewritten as

$$\int_\Omega EI\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2(\delta w)}{\partial x^2} - r^2\frac{\partial^3 w}{\partial x^3}\frac{\partial^3(\delta w)}{\partial x^3}\right)dx + \int_\Omega \rho A\frac{\partial^2 w}{\partial t^2}\delta w dx = 0. \tag{11}$$

We can assume that

$$w = W(x)e^{i\omega t}, \tag{12}$$

where $W(x)$ is the displacement at x. Substituting Eq. (12) into Eq. (11) gives

$$\int_\Omega EI\left(\frac{d^2 W}{dx^2}\frac{d^2(\delta W)}{dx^2} - r^2\frac{d^3 w}{dx^3}\frac{d^3(\delta W)}{dx^3}\right)dx - \omega^2\int_\Omega \rho AW\delta W dx = 0. \tag{13}$$

Since the MLS approximation can automatically satisfy the higher-order continuity, the components $W_{,xx} = \frac{d^2W}{dx^2}$ and $W_{,xxx} = \frac{d^3W}{dx^3}$ in Eq. (13) can be approximated with nodal components directly. The calculation of shape functions and their derivatives by the MLS method are illustrated as follows. The deflect $W(x)$ defined on the segment $[0, L]$ is set as

$$W^M(x) = \sum_{i=1}^m p_i(x)a_i(x) = \mathbf{p}^T(x)\mathbf{a}(x), \tag{14}$$

where $p_i(x)$ are the monomial basis functions, $a_i(x)$ are the coefficient of the basis functions and m is the number of terms in basis function. To construct the third-order derivative, the follow cubic basis function is used:

$$\mathbf{p}^T(x) = \left(1, x, x^2, x^3\right). \tag{15}$$

The unknown coefficient $a_i(x)$ in Eq. (14) can be determined by the minimization of the weighted residual J

$$J = \sum_{j=1}^{NP} G(x - x_j)\left[\mathbf{p}^T(x_j)\mathbf{a}(x_j) - W_j\right]^2, \tag{16}$$

where $G(x - x_j)$ is the weight function, and NP is the number of nodes within $G(x - x_j) > 0$. The minimum of J in Eq. (16) with respect to $\mathbf{a}(x)$ leads to a set of linear equations:

$$\mathbf{A}(x)\mathbf{a}(x) = \mathbf{B}(x)\mathbf{W}, \tag{17}$$

where $\mathbf{W} = (W_1, W_2, \cdots, W_{NP})^T$, and

$$\mathbf{A}(x) = \sum_{j=1}^{NP} G(x - x_j)\mathbf{p}(x_j)\mathbf{p}^T(x_j), \tag{18a}$$

$$\mathbf{B}(x) = \left[G(x - x_1)p(x_1), G(x - x_2)p(x_2)\cdots, G(x - x_{NP})p(x_{NP})\right]. \tag{18b}$$

Thus, the coefficients $a(x)$ can be obtained from Eq. (17) as

$$\mathbf{a}(x) = \mathbf{A}^{-1}(x)\mathbf{B}(x)\mathbf{W}. \tag{19}$$

Substituting Eq. (19) into Eq. (14), one can get

$$W^M(x) = \mathbf{p}^T(x)\mathbf{A}^{-1}(x)\mathbf{B}(x)\mathbf{w} = \boldsymbol{\varphi}\mathbf{w} = \sum_{j=1}^{NP}\phi_j(x)W_j(x), \tag{20}$$