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Efficient analysis of plates on nonlinear foundations

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ABSTRACT

This paper presents efficient analysis of plates on nonlinear foundations. The Reissner plate theory is used to model plates. Foundations are presented as the Winkler springs or the elastic half space. The developed analysis is mainly presented for tensionless foundation; however as demonstrated, it is straightforward extended to analysis of elastic-plastic foundations. The plate is analyzed using the boundary element method (BEM). Unlike the traditional BEM which uses equations in form ([H] {u} = [G] {t}), the presented formulation uses finite element like equations, in the form of ([K] {u} = {P}). An innovative formulation is presented to derive the relevant plate stiffness matrix [K] and load vector {P} from the BEM integral equation. Iterative procedures together with condensation process are used to eliminate degree of freedom at failed zones. Results of the present analysis are more accurate than those obtained from previously published results. The main advantages of the presented technique are its simplicity and accuracy and it gains both advantages of the boundary element and the finite element methods.

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1. Introduction

Plates on nonlinear (tensionless or elastic-plastic) foundations are an important problem in mechanics and have several applications in practical structural and geotechnical engineering. Numerical analysis of plates over elastic foundations (Winkler springs or elastic half space) is considered by many authors as follows:

(a) Using the finite element method: the work of Cheung and Zienkiewicz [2], Svec and McNeice [3], Svec and Gladwel [4] for thin plates, Rajapaks and Selvadurai [6] for thick plates.
(b) Using the boundary element method: the work of Katsikadelis and Armenakas [5], Syngellakis and Bai [7], Paiva and Butterfield [8] for thin plates, Rashed et al. [9–12] for thick plates.

Solving plates on tensionless foundation can be categorized into two main categories. The first category involves solution using iterative procedure to consider the miscontact between plate and foundation. The second category, on the other hand, involves solution by transforming the problem into a set of nonlinear equations, which could be solved using optimization algorithms.

Among the previous literature which follows the first category are: the work of Cheung and Nag [13] who presented a finite element analysis of beams and plates on linear and nonlinear elastic continuum. Weitsman [14] presented an approximate solution for the radius of contact between an elastic plate and a semi-infinite elastic half space subjected to concentrated load. Weitsman [15] presented analysis of tensionless beams, or plates, and their supporting Winkler or Reissner foundations due to concentrated loads. Svec [16] developed a finite element iterative procedure to determine the contact region between the plate and the elastic half space. The continuous contact pressure is approximated in [16] by a set of statically equivalent forces acting at the nodal points of the elements. Hence, the plate could be considered to be resting on a complicated system of springs. Celep [17] presented the behavior of elastic plates of rectangular shape on a tensionless Winkler foundation using auxiliary function. Galerkin's method is used in [17] to reduce the problem to a system of algebraic equations. Li and Dempsey [18] used an iterative procedure to analyze unbonded contact of a square thin plate under centrally symmetric vertical loading on elastic Winkler or elastic half space foundations. Hu and Hartely [19] solved thin plate on tensionless elastic half space using integral equations. Analysis using the T-element of plates on unilateral elastic Winkler type foundation is presented by Jirousek et al. [20]. Nonlinear bending behavior of Reissner–Mindlin plates with free edges resting on tensionless elastic foundations of the Pasternak-type using admissible functions is presented by Hui-Shen and Yu [21]. Results of finite element analysis of beam elements on unilateral elastic foundation using special zero thickness element designed for foundation modeling is presented by Torbacki [22]. Buczkowski and Torbacki [23] presented finite element analysis of plate on layered tensionless foundation. Kongtng and Sukawat [24] used the method of finite Hankel integral transform to solve the mixed boundary value problem of unilaterally supported rectangular plates loaded by uniformly distributed load.

The studies based on the second category, on the other hand, could be listed as follows:

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Sapountzakis and Katsikadelis [25] presented boundary element solution for unilateral contact problems of thin elastic plates resting on linear or nonlinear subgrade by solving a system of nonlinear algebraic equations. Xiao et al. [26] presented a coupled boundary element local complementary equation method (BE-LCEM) to solve thin free edge plates on elastic half space with unilateral contact. Silva et al. [27] used the finite element method to discretize the plate and foundation. Hence they used three alternative optimization techniques to solve plates on tensionless elastic foundations. Xiao [28] presented a BE-LCEM solution to solve unilateral free edges thick plates. Nonlinear analysis of structural elements under unilateral contact constrains studied via Ritz approach using mathematical programming technique is presented by Silveira et al. [29].

Among the previous literature review, the following three important works are re-considered herein in more details as they will be considered for the sake of comparison in Section 6 in this paper:

(1) The work of Hu and Hartely [19] in 1994:
They used the boundary element method to solve thin plates on tensionless elastic half space. In their formulation the flexibility equations obtained from the BEM ([H] {u} = [G] {t}) is directly coupled to the flexibility equations of the elastic half space. Unilateral contact between plate and elastic half space is solved using iterations by eliminating the tensile stress values. This was carried out by removing the row and column of the flexibility matrix that corresponds to the relevant DOF. It has to be noted that this removal operation is suitable for flexibility-like equations; however, as will be demonstrated in this paper (Section 3), this elimination procedure is carried out using condensation which is suitable for stiffness-like equations ([K] {u} = {P}). In addition Hu and Hartely formulation in Ref. [19] demonstrated difficulty in computing bending moments in vicinities of concentrated loads, which will not be the case in the presented formulation.

(2) The work of Silva et al. [27] in 2001:
Silva et al. [27] used the finite element method to solve thin or thick plates on tensionless foundations via optimization algorithms. They presented only one example that solves plate on tensionless elastic half space. This example was previously solved in Hu and Hartely [19]. Only displacements along centerline were compared and no bending moments results were reported. This example will be presented in Section 6.4 for the purpose of comparison.

(3) The work of Xiao [28] in 2001:
Xiao [28] solved unilateral free edges thick plates using the boundary element method. He presented the fundamental solution in terms of two Hu's potential functions. This leads to have high order singularities in his formulation; therefore he used a regular collocation scheme to avoid treatment of singularities that appeared in his formulation. It will be demonstrated in Sections 6.3 and 6.5 in this paper this leads to loose of accuracy near corners.

As previously demonstrated the available examples that consider solution of the plates on elastic half space considering nonlinearities is rare. However, there are few available finite element based software packages that could model half space as 3D solid elements. Despite the huge computational cost for such models, they give somehow good representation of the half space. Moreover such models are feasible for small test cases; therefore in Section 6.5 in this paper such a model is used to compare contact zones, deflections, foundation pressures and bending moments.

Generally, in this paper an iterative procedure (follows the first category) is developed to solve thick plates (as a more general theory that can model thin and thick plates [9–12]) on tensionless Winkler or elastic half space foundation. The boundary element method is used, in an innovative way, to extract the stiffness matrix and the load vector of the plate. The Mindlin's equations [30] are used to compute the elastic half space stiffness matrix. A coupling technique is used to assembly the overall stiffness matrix and load vector of the problem. The main advantage of the proposed technique is it inherits advantages of the boundary element method in modeling plate using integral equation and the simplicity of finite element like procedures in the coupling between plate and foundation. The proposed technique is simple, practical and it is suitable to be extended to include elastic-plastic analysis as will be presented in this paper (Section 4). Numerical examples are presented to verify the efficiency and practicality of the proposed technique.

2. Proposed boundary elements for plate on elastic foundation

In this section, the direct boundary element formulation of thick plates is reviewed. Hence an innovative derivation of plate stiffness matrix and load vector is presented. Finally a coupling technique between the plate and the foundation stiffness matrices is presented.

2.1. Boundary element for plates

Consider a general thick plate [1] of domain Ω and boundary Γ with internal stiffness cells as shown in Fig. 1. The stiffness cells are defined as supporting cells such as supporting columns, walls, piles, or foundation reactions. Each stiffness cell has three unknown reactions (two moments and one shear). The indicial notation is used in this paper where the Greek indexes vary from 1 to 2 and Roman indexes vary from 1 to 3. The relevant integral equation can be rewritten as follows [11]:

$$\begin{aligned} C_{ij}(\xi)\, u_j(\xi) + \int_{\Gamma(x)} T_{ij}(\xi,x)\, u_j(x) d\Gamma(x) = \int_{\Gamma(x)} U_{ij}(\xi,x)\, t_j(x) d\Gamma(x) \\ + \int_{\Gamma(x)} \left[V_{i,n}(\xi,x) - \frac{\nu}{(1-\nu)\lambda^2} U_{i\alpha}(\xi,x) n_\alpha(x) \right] q_3(x) d\Gamma(x) \\ + \sum_{N_q} \int_{\Omega(L)} \left[U_{ik}(\xi,L) - \frac{\nu}{(1-\nu)\lambda^2} U_{i\alpha,\alpha}(\xi,L) \delta_{3k} \right] P_k(L) d\Omega(L) \\ + \sum_{N_{sc}} \int_{\Omega(y)} \left[U_{im}(\xi,y) - \frac{\nu}{(1-\nu)\lambda^2} U_{i\alpha,\alpha}(\xi,y) \delta_{3m} \right] F_m(y)\, d\Omega(y) \end{aligned} \quad (1)$$

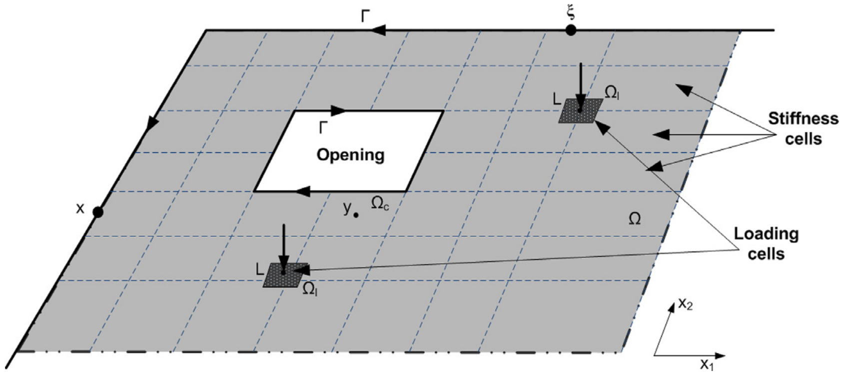

Fig. 1. Plate geometry and definitions.

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