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An adaptive meshless parameterization for full waveform inversion

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ABSTRACT

Full waveform inversion is a technique to recover images of the subsurface using data from a seismic survey. Nonlinearity, ill-posedness, presence of noise in data, a large number of parameters, different kinds of parameters, and limitations of data are some of the factors that contribute to instabilities of the solution. One of the strategies to regularize the solution is a suitable choice of parameterization. Depending on the parameterization strategy, the solution is searched in a space with certain features, for instance, smoothness, that may be convenient to the problem. Furthermore, a parameterization that can represent well the solution with a reduced number of parameters may be robust due to the limited number of degrees of freedom. Here we show an adaptive meshless parameterization methodology for full waveform inversion, which may also be useful for other inverse problems. The parameterization is based on a meshless technique and uses Wendland's functions as basis functions to an interpolation. As a meshless method, it is very flexible, then the spatial distribution of the unknowns can be non-uniform, allowing focusing on certain areas or automatically improving the quality of the image near the discontinuities of a velocity model. With some numerical experiments of acoustic inversion of synthetic data, we show that the parameterization methodology described here can represent complex velocity models and that, with a reduced number of unknowns, the inversion process behaves better than the standard parameterization with blocks. We also show that the adaptive meshless parameterization has a significant regularization effect, avoiding non-natural patterns and prioritizing smooth images.

1. Introduction

Full waveform inversion is a technique to recover an image of the subsurface using data from a seismic survey. It is computationally expensive because it is based on modeling of wave propagation. Although the first concepts of the technique were stated in the 80's [1], the progressive improvement of computational resources made the technique viable. Nowadays, a lot of studies have contributed to this important and promising tool for exploration geophysics.

It is well known that many aspects are involved in solving inverse problems. The algorithms can generate inaccurate solutions due to noise in data, sensitivity of different kinds of parameters, and rough parameterization or over-parameterization. In this paper, parameterization means the description of a physical system by a set of **m** parameters [2].

One of the regularization strategies is called multiscale inversion [3]. In this approach, from a smooth initial model, low-frequency components of data are inverted, generating an updated image. Then, such image is used as input to the same procedure, but with a higher frequency. This is repeated while the stop criterion is not reached. The multiscale approach can be considered a regularization strategy, since regularization means to give preference to models that reflect prior knowledge or expectation, ensuring the convergence towards physically meaningful models [4].

Moreover, regularization can be reached by choice of the parameterization as long as the solution has features that depend on the chosen basis functions [4]. For instance, Debayle and Sambridge [5] have presented a parameterization based on cells generated by optimized Voronoy diagrams. Nolet [6] has proposed a minimum energy parameterization based on blocks distributed non-uniformly depending

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on the resolution expected for each region of the Earth. Wang et al. [7] have proposed refinement guidelines to discretize in different scales an inverse electrocardiographic problem, addressing the ill-posedness. Boehm and Ulbrich [8] have used different meshes for the modeling and the parameter space for full waveform tomography because a coarser mesh in the parameter space prevents an over-parameterization. Furthermore, they combine regularization by discretization and multi-frequency inversion to achieve stability. Bangerth [9] presented an adaptive finite element solution to solve efficiently inverse problems in an all-at-once approach. The main feature of the methodology is the use of a continuous setting to allow discretizations that are adaptively refined as nonlinear iterations proceed. Peters and Barra [10] presented a method to parameterize the electrical impedance tomography problem, in which the parameters are the coordinates of control points and shape variables of X-splines. Valente et al. [11] have presented a methodology to solve the 3D wave equation using finite volumes in which space is discretized in a non-structured OcTree mesh [12] that allows better representation of the physical model regarding geometry and spatial distribution of the wavelength, avoiding the excessive refinement required by structured meshes.

A parameterization can use local or global basis functions to approximate the model. Local functions are defined non-null in their influence region and are null outside the region. Global functions are defined throughout the entire model. For full waveform inversion, the use of local functions is suitable since sharp contrasts in physical properties are expected. Moreover, it is common to define one parameter for each unknown of the numerical method used to solve the forward problem [4]. However, in this case, the number of parameters can be enormous and, depending on the problem, making the inversion process much more computationally expensive. Furthermore, the model can have too much freedom to fit noise in data, increasing the instability of the inversion process [4]. In conclusion, the spatial discretization of the forward problem depends on the stability criteria of the numerical method and the suitable representation of the physical properties. On the other hand, the parameterization depends on the resolution and the size of the structures that are expected to be resolvable.

Therefore, we propose a parameterization methodology that deals with different degrees of freedom for the forward problem and inverse problem. This parameterization is based on a particular set of basis functions called Wendland's functions [13]. These functions can be used to generate 2D and 3D models with specific continuity classes, this is, smoothness levels. Each basis function has one basis point. To each basis point, there is an associated parameter. The set of basis points is distributed uniformly or non-uniformly in the model, as a cloud of points, which allows increasing the resolution in regions of interest. Moreover, in order to be able to represent the physical properties, the region influenced by each basis function must be larger than in the standard block parameterization. So, the meshless parameterization leads to a strong relation between two neighbor nodes because of the smooth interpolation function. A small change in a parameter affects a region larger than in block parameterization, increasing the stability of the solution.

In order to avoid excessive smoothing near discontinuities of the physical model, we suggest a strategy that distributes the basis points non-uniformly. Such strategy is based on the "spring analogy" [14] and tries to concentrate more unknowns near regions of interest. The distribution follows a rule that takes into account, for each imaging frequency, the spatial variation of the wavelength.

In this paper, we present some numerical experiments showing that the proposed parameterization is able to represent complex models of physical properties, generating images with high structural similarities with the true model.

2. Methods

Our main objective is to show that it is possible to generate suitable images using a number as small as possible of parameters. Depending on the problem, it could lead to the reduction of the required memory and the number of operations required. In the context of seismic inversion, any contribution in this issue is important in the case of 3D imaging and multiparameter inversion. Another feature is to increase the robustness of the inversion methodology because the use of a small number of degrees of freedom can be a regularization strategy. On the other hand, the use of a limited number of parameters requires interpolation to generate velocity models, which can excessively smooth the interfaces between regions with different physical properties.

Another point is related to the discretization required by the numerical method to satisfy its stability criteria. Very often, such discretization is finer than that required by the image resolution. For example, some Finite Difference (FD) schemes need a grid spacing of $\lambda/5$ or $\lambda/10$, while the expected resolution of the imaging is $\lambda/2$, where λ is the wavelength. So, the use of the same discretization for the forward and inverse problem is an over-parameterization case and could lead to inappropriate solutions that can be improved by regularization strategies, like Tikhonov or total variation regularization.

To make the discretization of the forward and inverse problem independent regarding the number of degrees of freedom and kind of mesh, we use a meshless parameterization. Then, the same parameters can be used to generate models of structured or unstructured meshes for different numerical methods: Finite Differences [15], Continuous or Discontinuous Finite Elements [16] and coupled methods [17]. Moreover, the methodology is suitable to provide focus in a particular region even when the numerical method employs uniform meshes.

Here we propose the use of a meshless parameterization with interpolation based on a Wendland's basis function [13]. Such function seems to be very suitable for inverse problems because the only parameter is the radius of support. In the context of the multiscale approach, for each imaging frequency, we compute the number of parameters to be used. The section Adaptivity shows the procedure to change the number of parameters.

Each parameter is related to a basis point. The basis points are distributed all over the model. This distribution can be uniform or non-uniform. As we have mentioned, interpolation tends to smooth the interfaces or discontinuities of the velocity model. However, if we approximate two basis points to the discontinuity, the smoothing is reduced and the image near the interface becomes better than when the points are far from the discontinuity. So, here we propose a methodology to spread the basis points based on an automatic identification of the interfaces. The idea is to concentrate more basis points near the interfaces, reducing the density of basis points over homogeneous areas. Here we propose the use of the "spring analogy method" to guide the non-uniform distribution of the basis points, what is explained in section "Spring Analogy".

In order to analyze the benefits of the non-uniform meshless parameterization, we compare three different strategies, summarized as follows:

- Blocks: The classical constant by parts parameterization. All the blocks have the same size.
- Uniform meshless: The parameterization is based on interpolation of Wendland's functions. The basis points of interpolation are uniformly distributed. With this strategy, we expect to generate smooth velocity models.
- Non-uniform meshless: Using the spring analogy, the basis points are distributed to be closer to the interfaces. With this strategy, we expect to avoid excessive smoothing of the interfaces when a reduced number of parameters is used.

The following sections describe the numerical solution of the forward problem, the mapping between parameters and velocity model, the adaptivity and the spring analogy.