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Stress calculation and optimization in composite plates with holes based on the modified integral equation method

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ABSTRACT

The integral equations are written for multiple connected anisotropic plates in a simple form based on the simple dependencies between the Lekhnitskii complex potentials and stress and strain. The numerical method for solving integral equations is developed by the mechanical quadrature method for systems of holes, which takes into account their eigen-solutions. Simplicity, precision of the approach and stability of obtained algebraic equations is illustrated in determination of hole shapes with low stress concentration; study of high stress concentration at slit of arbitrary width (additionally used asymptotic method); calculation of the stress with controlled accuracy for a large number of holes.

1. Introduction

The boundary integral equation method (BIEM) is widely used for studying the stress–strain state (SSS) of isotropic and anisotropic plates with holes and cracks [1–9].

The simplicity of algorithms that are developed based on BIEM for isotropic materials has led to their wide application as for the direct calculation of stress and also for solving nonclassical problems. In particular, the problem of determination of the optimal (with low stress concentration) shape of holes in which additional optimization methods are used is important for practical applications. This problem is particularly urgent for anisotropic materials because high stress concentration occurs at holes. Recently, for isotropic materials novel approaches were developed, which allow to calculate high localized stress concentrations. To implement this approach it is necessary to calculate the quick change in stress with a high accuracy and additionally to use asymptotic methods [10].

The integral equations for anisotropic plates are significantly more complex than those for isotropic materials. In [1,11] there are established basic relationships between the Lekhnitskii potentials and stresses and displacements and based on them and Cauchy theorem the integral equations are written in a simple form for anisotropic plates with cracks. Let us use the obtained relations for the derivation and regularization of integral equations to multiple connected plates. Let us illustrate the effectiveness and simplicity of use of the developed numerical algorithm to optimize the shape of the hole and to build high precise formulas to determine the concentration of stresses near the slit with arbitrary width.

2. Formulation of the problem

The plate is related to the Cartesian coordinate system Oxy. Assume that a plate is loaded with tractions (X_L, Y_L) that are applied to its boundary. Moreover $X_L = X_L(x,y)$, $Y_L = Y_L(x,y)$ are given functions at $(x, y) \in L_j$, $j = 0, \ldots J$, the applied load is in self-equilibrium (Fig. 1a).

It is also considered a plate of infinite size (the path L_0 placed at infinity). In this case, instead of the conditions at the path L_0 it is assumed that plate is in a condition of bilateral tension at infinity (Fig. 1b). Let us denote the domain occupied by the plate by D.

3. Governing equations

Let us start from Lekhnitskii complex potentials $\Phi(z_1)$, $\Psi(z_2)$, where $z_j = x + s_j y$, and s_j, $j = 1, 2$ are roots with positive imaginary part of characteristic equation $\Delta(s) = 0$ [12], where

$$\Delta(s) = \alpha_{11}s^4 - 2\alpha_{16}s^3 + (2\alpha_{12} + \alpha_{66})s^2 - 2\alpha_{26}s + \alpha_{22} = 0, \quad (1)$$

α_{ij} are elastic compliances which are included in the Hooke's law [12]

$$\varepsilon_x = a_{11}\sigma_x + a_{12}\sigma_y + a_{16}\tau_{xy}, \ \varepsilon_y = a_{12}\sigma_x + a_{22}\sigma_y + a_{26}\tau_{xy}, \gamma_{xy} = a_{16}\sigma_x + a_{26}\sigma_y + a_{66}\tau_{xy}$$

where ε_x, ε_y, γ_{xy} are strains, σ_x, σ_y, τ_{xy} are stresses.

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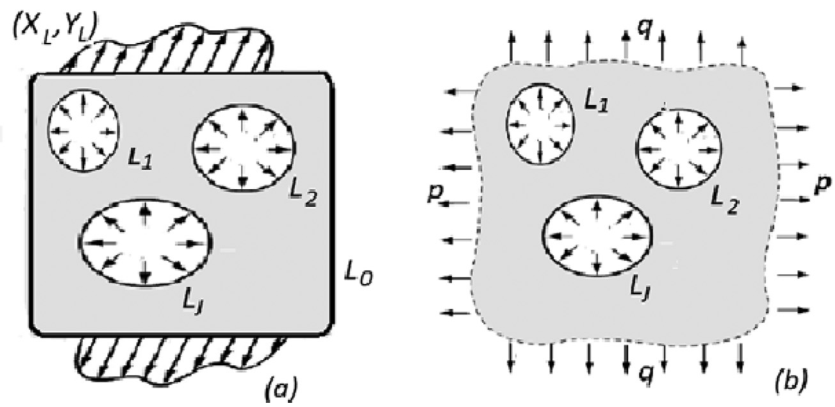

Fig. 1. The composite plate weakened with a system of holes: the plate with the outer boundary (a); the plate of infinite size (b).

Consider an arbitrary path Γ, which belongs to the domain D occupied by the plate.

The tractions $(X,\ Y)$ and displacements $(u,\ v)$ are determined at this path by the formulas [11,12]

$$Y = -2\mathrm{Re}\left[\Phi(z_1)z_1' + \Psi(z_2)z_2'\right],\quad X = 2\mathrm{Re}\left[s_1\Phi(z_1)z_1' + s_2\Psi(z_2)z_2'\right],$$

$$u' = 2\mathrm{Re}\left[p_1\Phi(z_1)z_1' + p_2\Psi(z_2)z_2'\right],\quad v' = 2\mathrm{Re}\left[q_1\Phi(z_1)z_1' + q_2\Psi(z_2)z_2'\right], \tag{2}$$

where $u' = du/ds$, $v' = dv/ds$, $p_j = \alpha_{11}s_j^2 - \alpha_{16}s_j + \alpha_{12}$, $q_j = \alpha_{12}s_j - \alpha_{26} + \alpha_{22}/s_j$, $z_j' = dx/ds + s_j dy/ds$, ds is a differential of arc at Γ.

Then introduce in consideration the stress vectors $q_\Gamma(z) = X + iY$ at path Γ, which is determined on this path by the formula

$$q_\Gamma = (s_1 - i)z_1'\Phi(z_1) + (\overline{s_1} - i)\overline{z_1'}\,\overline{\Phi(z_1)} + (s_2 - i)z_2'\Psi(z_2) + (\overline{s_2} - i)\overline{z_2'\Psi(z_2)}. \tag{3}$$

Assume that the vectors $(X,\ Y)$ and $(u,\ v)$ are knows at the path Γ. Then based on (2) one has [11]

$$\Phi(z_1) = \frac{-v' + s_1u' + p_1X + q_1Y}{\Delta_1 z_1'},\quad \Psi(z_2) = \frac{-v' + s_2u' + p_2X + q_2Y}{\Delta_2 z_2'}, \tag{4}$$

where $\Delta_j = \Delta'(s_j)$, $j = 1, 2$.

Before construction of the problem solution additionally assume that at some point M of the plate displacement along the axes and rotation are given in advance (i.e. zero). Then according to existence theorem for considered problem the solution exists and it is unique regarding stresses and displacements. These uniquely defined displacements are denoted through u, v.

Let us build a general solution to the problem. For this we use the Lekhnitskii function $\Phi(z_1)$ is analytic for the variable $z_1 = x_1 + iy_1$ at the domain D' which is obtained from the domain D as a result of Lekhnitskii affine transformations $x_1 = x + y\mathrm{Re}s_1$, $y_1 = y\mathrm{Im}s_1$. Then by using Cauchy theorem, we get

$$\Phi(z_1) = \frac{1}{2\pi i}\int_{L'}\frac{\Phi_L(t_1)dt_1}{t_1 - z_1},$$

where L' is the boundary of the domain D', Φ_L is the function $\Phi(z_1)$ at the path L', $t_1 \in L'$.

With uniquely defined displacements u, v at the boundary of the plate function Φ_L is uniquely determined by the first Eq. (4). Then we get

$$\Phi(z_1) = \frac{1}{2\pi i}\int_{L'}\frac{s_1u' - v' + p_1X_L + q_1Y_L}{\Delta_1 dt_1/ds}\frac{dt_1}{t_1 - z_1},$$

where $t_1 \in L'$, L' is the boundary of the domain D', u', v' are derivatives of displacements (unknown) on the boundary curves L_j, $j = 0, \ldots, J$.

There takes into account that the values X, Y in the formulas (4) at the boundary equal to given efforts X_L, Y_L, respectively, and the integration direction at the boundary path is selected by Cauchy theorem such that these curves while traversing domain D' are to the left.

Similarly, we determine the potential Ψ

$$\Psi(z_2) = \frac{1}{2\pi i}\int_{L''}\frac{\Psi(t_2)dt_2}{t_2 - z_2} = \frac{1}{2\pi i}\int_{L''}\frac{s_2u' - v' + p_2X_L + q_2Y_L}{\Delta_2 dt_2/ds}\frac{dt_2}{t_2 - z_2},$$

where L'' is the boundary of the domain L'', which is obtained from the domain D under affine transformations $x_2 = x + y\mathrm{Re}s_2$, $y_2 = y\mathrm{Im}s_2$, $z_2 = x_2 + iy_2$, $t_2 \in L''$.

Hence we obtain integral representations for potentials, which can be written as

$$\Phi(z_1) = \int_L\left[u'(s)\Phi_1(z_1, t_1) + v'(s)\Phi_2(z_1, t_1)\right]ds + \Phi_\Delta(z_1),$$

$$\Psi(z_2) = \int_L\left[u'(s)\Psi_1(z_2, t_2) + v'(s)\Psi_2(z_2, t_2)\right]ds + \Psi_\Delta(z_2), \tag{5}$$

where s is an arc coordinate at the path L, $L = L_0 + L_1 + \ldots + L_J$, $ds = \sqrt{(d\xi)^2 + (d\eta)^2}$, $t_k = \xi + s_k\eta$, $k = 1, 2$, $(\xi,\ \eta) \in L$,

$$\begin{aligned}\Phi_\Delta(z_1) &= \frac{1}{2\pi\Delta_1 i}\int_L\frac{p_1X_L(s) + q_1Y_L(s)}{t_1 - z_1}ds\\ &= \int_L\left[X_L(s)\Phi_3(z_1, t_1) + Y_L(s)\Phi_4(z_1, t_1)\right]ds,\\ \Psi_\Delta(z_2) &= \frac{1}{2\pi\Delta_2 i}\int_L\frac{p_2X_L(s) + q_2Y_L(s)}{t_2 - z_2}ds\\ &= \int_L\left[X_L(s)\Psi_3(z_1, t_1) + Y_L(s)\Psi_4(z_1, t_1)\right]ds,\end{aligned} \tag{6}$$

where $\Phi_j(z_1, t_1) = \frac{A_j}{t_1 - z_1}$, $\Psi_j(z_2, t_2) = \frac{B_j}{t_2 - z_2}$, $j = 1, \ldots, 4$,
$A_1 = \alpha_1 s_1$, $A_2 = -\alpha_1$,
$A_3 = \alpha_1 p_1$, $A_4 = \alpha_1 q_1$, $B_1 = \alpha_2 s_2$, $B_2 = -\alpha_2$, $B_3 = \alpha_2 p_2$, $B_4 = \alpha_2 q_2$,
$\alpha_k = \frac{1}{2\pi\Delta_k i}$, $k = 1, 2$.

Note that $\Phi_\Delta(z_1)$, $\Psi_\Delta(z_2)$ are known functions that can be found in particular cases in an analytic form, and in the case of arbitrary distribution of load by using quadrature formulas.

4. The integral equation of the problem

For writing the integral equations it is necessary to determine the stresses, which correspond to representations (4) at the boundary of the plate and equate them to the given ones. For this determine the potential