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A continuous–discontinuous hybrid boundary node method for solving stress intensity factor

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ABSTRACT

A novel boundary type meshless method called continuous–discontinuous hybrid boundary node method is proposed in this paper, in which the enriched discontinuous shape function is developed to solve linear elastic crack problems. Firstly, the whole boundary is divided into several individual segments, and variables on each one of those segments are interpolated, respectively. For continuous segments, radial point interpolation method is employed. In regard to discontinuous segments, the enriched discontinuous basis functions combining with radial point interpolation method are developed for simulating the discontinuity of displacement and stress field on surfaces of crack, and the near tip asymptotic field functions are employed for simulating the high gradient of stress field around crack tip, so that high accuracy and discontinuity property of a crack can be easily described. Stress intensity factors are calculated directly using displacement extrapolation by displacement field near crack tip. Some numerical examples are shown that the present method is effective and can be widely applied in some practical engineering.

1. Introduction

For many structures, crack propagation is an important failure mechanism and requires accurate simulation essential for failure prediction. And stress intensity factors are important property, which are directly related to fracture propagation criteria. The analytical solutions of stress intensity factors are hardly got for many complex structures, so numerical methods must be applied for many cases.

Now many analytical and numerical methods are applied to solve fracture response and reliability of cracked structures. The most popular method recently is the finite element method (FEM). Although FEM is useful for many engineering analysis, even for fracture analysis of cracks, the method has serious limitations in some problems characterized by a continuous change in geometry of the domain under analysis. Crack propagation is a prime example in which a large number of remeshing is needed in the use of FEM. For FEM and some similar methods, the only viable option for dealing with moving cracks is remeshing during each discrete step, which is cumbersome and time-consuming. The boundary element method (BEM) [23] and dual boundary element method (DBEM) [24,25] which have certain advantages over FEM has been also applied to solve crack problems in past decades, but the same as FEM, element is inevitable in the calculation.

Meshfree or meshless methods have been developed rapidly in recent years. A class of meshfree and meshless methods appear to demonstrate significant special for the moving boundary problems typified by crack propagation. Such as smooth particle hydrodynamics (SPH) [11,15], diffuse element method (DEM) [19], element-free Galerkin method (EFGM) [4,12], meshless local Petrov–Galerkin method (MLPGM) [3], local boundary integral method [2], and meshless singular boundary method(MSBM) [8,9]. For those methods, the elements meshing is not used, since only a scattered set of nodal points is required in the domain of interest. Since no element connectivity is needed, the burdensome remeshing required by FEM is avoided.

Though all meshless methods do not need the element meshing for field variable interpolation, some of them require a background meshing for integration. For example, the EFG method [4,12] uses moving least square (MLS) for the shape function interpolation, and it does not require element mesh for variable interpolation. However, background element is inevitable for integration.

Appling MLS to the boundary integration equations, Mukherjee and Mukherjee proposed boundary node method (BNM) [16], which only requires to discretize the boundary. Although this method does not require an element mesh for the interpolation of the boundary variables, a background element is still necessary for integration. Based on BNM, Zhang et al. [39,40] proposed another boundary-type meshless method: hybrid boundary node method (HBNM). It abandons background elements and achieves a truly meshless method. Elements are required neither for interpolation nor for integration. However, it has a drawback of serious 'boundary layer effect'.

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To avoid this shortcoming, Zhang and Yao further proposed the regular hybrid boundary node method [41,42], in which the source points of fundamental solution are located outside of the calculating domain. Although this method can avoid the singular integration and boundary layer effect, it creates some new problems. For example, how to arrange the positions of the source points? To overcome these problems, Wang et al. [30] presented a meshless singular hybrid node method for 2-D elasticity, and obtained satisfactory results by means of reasonable treatment of nearly singular integrals. And Miao et al. [13] proposed the rigid body motion approach to deal with singular integration and applied an adaptive integration scheme to solve boundary layer effect.

Those methods, however, can only be used for solving homogeneous problems. For inhomogeneous problems, domain integration is inevitable, and some methods are proposed for domain integral, for example, novel adaptive meshfree integration techniques are developed by Racz and Bui [26], besides, dual reciprocity method (DRM) was first proposed by Nardini and Brebbia [18] for elasto-dynamic problems in 1982 and extended by Wrobel and Brebbia [31] to time dependent diffusion in 1986. Based on HBNM, DRM is first introduced into HBNM, and a new truly meshless method dual hybrid boundary node method (DHBNM) is proposed by Yan et al. [36,38] which can be applied to inhomogeneous problems, dynamic problems and nonlinear problems and so on. Furthermore, based on radial point interpolation method and Taylor expansion, Yan et al. proposed a series of new hybrid boundary node methods [33,37], and by those methods some new shape function construction methods have been developed. Furthermore, combining radial point interpolation method, Yan et al. developed a new dual hybrid boundary node methods [34,35], later Yan et al. [32] developed a new shape function constructing method, named Shepard and Taylor interpolation method (STIM), by which no inversion operation is needed in the whole process of shape function constructing.

Unfortunately, the above methods based on a pure continuous theory are inadequate to describe such kinematic discontinuity of displacement field, and a discontinuity description in displacement field has been shown to be the necessity to describe the excessive discontinuous gap for two surfaces of a crack. Recently, a lot of continuous–discontinuous approaches have been proposed for solving those problems, for example, Simone et al. [28] and Oliver et al. [21] proposed a computational framework for the description of the continuous–discontinuous failure in a regularized strain-softening continuum; Oliver [20] employed strain softening constitutive equations to model strong discontinuities in solid mechanics; then Moes et al. [14] and Armero and Linder [1] and Stolarska et al. [29] developed the extended finite element method for analysis of crack growth, and so on.

DHBNM has some advantages, such as, dimensionality reduction, no element meshing, high accuracy and easily performance for large deformation, and it is widely used for inhomogeneous problems, nonlinear problems and dynamic problems, but its continuous property causes the difficult of application for crack propagation. Based on the boundary type meshless advantage property of hybrid boundary node method, a new continuous–discontinuous hybrid boundary node method is proposed to solve strong discontinuities problems in this paper. Firstly, the whole boundary is divided into several individual segments, and variables on each one of those segments are interpolated individually. For continuous segments, radial point interpolation method is employed. In regard to discontinuous segments, the enriched discontinuous basis functions combining with radial point interpolation method are developed for simulating the discontinuity of displacement and stress field on surfaces of fracture, and the near tip asymptotic field function is employed for simulating the singularity of crack tip stress field around the crack tip. Based on the above theory, a discontinuous hybrid boundary node method is proposed in this paper. And high accuracy and discontinuity property of a crack can be easily described in present method. This method keeps the ‘boundary-only’ and truly meshless method character of HBNM. The present work uses directly displacement extrapolation to calculate stress intensity factors. Besides, in order to simulate the singularity of stress field on the tip of fracture, and the enriched basis functions combined with radial point interpolation method are used near the tip of crack. Some numerical examples are shown that the present method is effective and can be widely applied in some practical engineering.

The discussions of this method arrange as following: the hybrid boundary node method will be discussed in Section 2. The enriched discontinuous interpolation is developed in Section 3. Some numerical implementation is demonstration in Section 4. The numerical examples for 2-D linear elastic crack problems are showed in Section 5. Finally, the paper will end with conclusions in Section 6.

2. Hybrid boundary node method

In this section, HBNM is introduced. Consider a 2D elasticity problem in domain Ω bounded by Γ. The basic equations are

$$\sigma_{ij,j} = b_i \tag{1}$$

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where b_i is the body force, λ is the Lame constant and G is the shear modulus.

The boundary conditions can be given as

$$u_i = \hat{u}_i \quad \text{on } \Gamma_u \tag{4}$$

$$\sigma_{ij}n_j = \hat{t}_i \quad \text{or } \Gamma_t \tag{5}$$

where $\hat{u}_i$ and $\hat{t}_i$ denote boundary node values and n is the unit outward normal to the domain boundary Γ.

Solving the above equations, the second-order partial differential equations for the displacement components can be obtained as [37]

$$Gu_{i,kk} + \frac{G}{1-2\nu}u_{k,ki} = b_i \tag{6}$$

Applying DRM, the solution variables u_i can be divided into two parts, i.e., the complementary solutions u_i^c and the particular solutions u_i^p, that is

$$u_i = u_i^c + u_i^p \tag{7}$$

The particular solution u_i^p has to satisfy the inhomogeneous equation as

$$Gu^p_{i,kk} + \frac{G}{1-2\nu}u^p_{k,ki} = b_i \tag{8}$$

On the other hand, the complementary solution u_i^c must satisfy the homogeneous equation and the modified boundary conditions. It can be written in the form [37]

$$Gu^c_{i,kk} + \frac{G}{1-2\nu}u^c_{k,ki} = 0 \tag{9}$$

$$u_i^c = \hat{u}_i^c = \hat{u}_i - u_i^p \tag{10}$$

$$t_i^c = \hat{t}_i^c = \hat{t}_i - t_i^p \tag{11}$$

where $\hat{u}_i^c$, $\hat{t}_i^c$ denote complementary solutions on boundary node i.