Engineering Analysis with Boundary Elements 80 (2017) 38–48

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

On modification of pressure gradient operator in integrated ISPH for multifluid and porous media flow with free-surface

Gourabananda Pahar, Anirban Dhar*

Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur, WB 721302, India

ARTICLE INFO

Keywords:
Incompressible smoothed particle hydrodynamics
Nonlinear flow through porous media
Density Current with low density ratio
Divergence-free ISPH

ABSTRACT

In Incompressible Smoothed Particle Hydrodynamics (ISPH) simulation, choice of pressure gradient operator plays an important role. Variations in effective porosity, fluid density, and free-surface conditions dictate the nature of the formulation. This study proposes an integrated ISPH framework with an implicit free-surface treatment. Pressure variation at the multi-fluid interface is maintained using a modified density-weighted pressure gradient with linear momentum conservation. Different pressure gradients with diffused interfaces for a porous domain as well as multifluid interface have been compared with the proposed operator. A unified form of Brinkman and Navier-Stokes equations are utilized to describe the flow-physics inside and outside the porous domain. Density variation in fluids is modelled by solving the scalar-transport equation. Effect of the porous domain is incorporated in terms of varying representative volume of the fluid particles. Porous media interface conditions are implicitly implemented using Darcy velocity and by introducing porosity into Pressure Poisson Equation (PPE). The present model is capable of minimizing error in velocity-divergence due to implicit free-surface treatment combined with linear momentum conservation. Proposed framework is validated by using existing experimental data of density-dependent flow with very low-density ratio and flow through porous blocks. A result of density-current passing through porous domain demonstrates the capability of the developed model for complex scenarios.

1. Introduction

Multifluid flow through porous domain problems is common in natural systems, e.g. saltwater intrusion in coastal aquifers. Saltwater intrusion occurs mainly due to the density difference between seawater and freshwater. These currents change their nature while passing through porous and non-porous phases. The gravity waves have a diffused interface between dense and light fluids. Solving multifluid flow through different phases requires proper modelling of the fluid front. A small-scale robust numerical framework is proposed here to simulate the density current through porous domains. The framework highlights the choice of pressure gradient operator in simulating miscible gravity currents as well as free-surface flow through porous media.

Smoothed Particle Hydrodynamics (SPH), a fully Lagrangian framework, was invented for simulation of astrophysical problems by Monaghan [1] and Lucy [2]. Being a particle-based method, it can readily track irregular boundary as well as the free surface. Initially, water was treated as a weakly compressible fluid by Monaghan [3]. A stiff artificial equation of state with sound speed was utilized to obtain fluid pressure. However, density renormalization and smoothing algorithm may be required to get smooth pressure variation. Later, divergence-free Incompressible SPH was developed by Cummins and Rudman [4]. Their method used a semi-implicit pressure projection scheme to maintain solenoidal velocity. Shao [5] solved incompressibility criteria using intermediate density. Integrated approaches of keeping density invariance and divergence free velocity scheme were suggested by Hu and Adams [6] and Asai et al. [7]. Density invariant ISPH frameworks may contain spurious numerical noises [8] and higher velocity-divergence errors [9].

Grenier et al. [10] resolved multifluid flow with large density difference using weakly compressible SPH (WCSPH). Shao [11] modelled immiscible multifluid flow using coupled and decoupled density-invariant ISPH for different density ratios. At the interface, the decoupled model searches for particles with same density. This model may not be suitable for a miscible fluid system, where density changes over time. Only a few studies have concentrated on SPH simulation of density current for fluids with low-density ratio, e.g., Ghasemi et al. [12]. However, they have not considered turbulence modelling or free-surface flow. Chen et al. [13] developed a framework

* Corresponding author.
E-mail addresses: gpahar@gmail.com (G. Pahar), anirban.dhar@gmail.com, anirband@civil.iitkgp.ernet.in (A. Dhar).

http://dx.doi.org/10.1016/j.enganabound.2017.02.015
Received 4 October 2016; Received in revised form 29 December 2016; Accepted 14 February 2017
0955-7997/

for maintaining pressure continuity at interface without directly using information of neighbouring particles. However, WCSPH models require various correction algorithm like density renormalization which may not be suitable for fluids with small density differences (≈1).

For free-surface flow and porous media interaction, Shao [14] proposed two sets momentum equation for inside and outside porous domain. Their method derived interaction conditions explicitly. Each time step modified the flow parameters based on an underlying grid. For heterogeneous porous media, this interface conditions may not be effectively maintained. Later, their methodology was improved by Pu and Shao [15] to a unified approach for momentum equations. However, the effect of porosity on representational volume was not taken into account by any of these works. A concept of apparent density for fluid particles inside porous media was introduced by Akbari et al. [16] based on porosity of the region. Particle volume is accordingly modified based on apparent density. A similar framework was developed by Aly and Asai [17] with predetermined relaxation coefficient for divergence of intermediate velocity in source/sink term of hybrid PPE. Ren et al. [18] proposed an improvement over weakly compressible SPH for modelling free-surface flow interaction with porous domain. Gui et al. [19] proposed averaging of particle pressures around interface using SPH kernel. These problems were mostly solved by using density-invariance scheme or its variant. For simulation of miscible fluids through porous media, actual density of fluid particles may change accordingly. Divergence-free velocity scheme is a better choice compared to Density Invariance scheme in those cases.

This study focuses on proper choice of pressure gradient operator for free-surface flow simulation using Incompressible SPH. Pressure gradient operators behave differently depending on density ratio and effective porosity and free-surface conditions. Miscible fluid particles may alter their density/mass according to scalar transport equation. Their representative volume also undergoes changes under the influence of effective porosity of the corresponding domain. Interface condition is solved implicitly through a unified set of governing equations. Dynamics of flow through porous media and lock exchange flow are investigated using the numerical framework. The integrated Incompressible SPH model for multi-fluid flow through a porous domain is compared with experimental data for different scenarios of flow through porous media and miscible lock exchange flow with very low-density ratio between saline and fresh water. A demonstrative example of integrated numerical simulation of density current through porous media underlines the capability of the model for complex flow situations.

2. Governing equations

Viscous Newtonian incompressible miscible multifluid flow in two-phase (fluid domain and porous domain) system is considered. The governing equations for simulating flow outside (Ω_f) and inside (Ω_d) the porous domain are conservation of mass and linear momentum equations. The unified governing equations [20] for continuity and momentum equation can be written as,

$$\nabla.\ \mathbf{u}_d = 0 \tag{1a}$$

$$\frac{D\mathbf{u}_d}{Dt} = -\frac{\eta}{\rho}\nabla P + \eta\mathbf{g} + \nu\nabla^2\mathbf{u}_d + \frac{\mathbf{R}}{\rho} + \frac{1}{\rho}\nabla.\ \overline{\tau} \tag{1b}$$

where $\mathbf{u}_d$=Darcy velocity vector, P=fluid pressure, ρ=fluid density, $\mathbf{g}$=acceleration due to gravity, ν=kinematic viscosity, $\overline{\tau}$=Sub-Particle-Scale (SPS) tensor, η=effective porosity, $\frac{\mathbf{R}}{\rho}$=resistive force. In the case of a pure fluid domain, Darcy velocity represents actual filtered velocity of the fluid. The resistive force vanishes for pure fluid domain to a filtered Navier-Stokes equation [21].

Scalar transport equation can be expressed as

$$\frac{DC}{Dt} = \Gamma\nabla^2 C \quad on\ \Omega_f \cup \Omega_d \tag{2}$$

where C=concentration, Γ=molecular diffusivity.

2.1. Resistive force

In Brinkman equation, the resistive force $\left(\frac{\mathbf{R}}{\rho}\right)$ can be expressed in terms of Darcy and Forcheimer drag forces [22]. For flow through finer porous media with low Reynolds number, the resistive force is limited to linear Darcy drag. However, Forcheimer drag increases with rising Reynolds number.

$$\frac{\mathbf{R}}{\rho} = -\left[\frac{\nu\eta^2\mathbf{u}}{k} + \frac{F_{ch}\eta^3\mathbf{u}|\mathbf{u}|}{\sqrt{k}}\right] \tag{3}$$

where $\mathbf{u}$=seepage velocity (actual fluid velocity in porous domain). Linear Darcy drag force depends on intrinsic permeability (k) of the porous media [22].

$$k = \frac{\eta^2 d_{50}^2}{\alpha(1-\eta)^2}$$

where d_{50}=the mean diameter of the porous domain, α=a problem specific variable (100–2000). Forcheimer parameter (F_{ch}) can be written as,

$$F_{ch} = \frac{1.75}{\sqrt{150\eta^3}}$$

These variables can be computed based on experimental and/or empirical approaches. Similar analytical/empirical formula have been utilized in [23,24]. The present Study utilizes the values given by Ghimire [22].

2.2. Turbulence modelling

Turbulence closure is required for resolving flow-physics [25,21] beyond particle scale in pure fluid domain. Present study utilizes semi-analytical Smagorinsky [26] model. Eddy viscosity is calculated by using large-scale flow velocity. Turbulence stress tensor ($\overline{\tau}$) can be formulated as,

$$\frac{1}{\rho}\overline{\tau}_{ij} = 2\nu_t S_{ij} - \frac{2}{3}\kappa\delta_{ij} \tag{4}$$

where ν_t=turbulence Eddy Viscosity, δ_{ij}=kronecker delta, S_{ij}=strain rate.

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Turbulence Eddy Viscosity (ν_t) can be determined as,

$$\nu_t = C_s^2\delta l^2|S| \tag{5}$$

where δl=characteristic length of small eddies/initial particle spacing, C_s=a constant. Local strain rate ($|S|$) can be calculated as,

$$|S| = (2S_{ij}S_{ij})^{0.5} \tag{6}$$

Turbulent kinetic energy (k) can be evaluated as,

$$\kappa = \frac{C_\nu}{C_\epsilon}\delta l^2|S|^2 \tag{7}$$

The constants are chosen as C_ν=0.08, C_ϵ=1.0, C_s=0.15. The filtered scalar transport equation can be written as per [27],

$$\frac{DC}{Dt} = \Gamma\frac{\partial^2 < C>}{\partial x_j x_j} - \frac{\partial < u'_j C'>}{\partial x_j} \tag{8}$$

where u'_j=fluctuation in velocity. <> represents filtered quantity of a parameter. Last term on the right side in Eq. (8) represents the sub-