Engineering Analysis with Boundary Elements 80 (2017) 116–126

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A stochastic perturbation edge-based smoothed finite element method for the analysis of uncertain structural-acoustics problems with random variables

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ARTICLE INFO

Keywords:
Stochastic analysis
Edge-based smoothed finite element (ES-FEM)
SP-ES-FEM
Uncertainties
Structural-acoustics problems

ABSTRACT

Among the current methods in predicting the response of structural-acoustics problems in mid-frequency regime, some problems such as low accuracy and inability to deal with the uncertainties still need to be solved. To eliminate these issues, a novel stochastic perturbation edge-based smoothed FEM method (SP-ES-FEM) is proposed for the analysis of structural-acoustics problems in this work. The edge-based smoothing technique is applied in the standard FEM approach to soften the over-stiff behavior of structural-acoustics problems aiming to improve the accuracy of deterministic response predictions. Then, this approach, for the first time, intends to introduce the first-order perturbation technique into the edge-based smoothed FEM theory frame especially for the probabilistic analysis of structural-acoustics problems. The response of the coupled systems can be expressed simply as a linear function of all the pre-defined input variables by using the change of variable techniques. Due to the linear relationships of variables and response, the probability density function and cumulative probability density function of the response can be obtained based on the simple mathematical transformation of probability theory. The proposed approach not only improves the numerical accuracy of deterministic output quantities with respect to a given random variable, but also can handle the randomness well in the systems. Two numerical examples for frequency response analysis of random structural-acoustics are presented and verified by Monte Carlo simulation, to demonstrate the effectiveness of the present method.

1. Introduction

The structural-acoustics problems are commonly encountered in numerous engineering systems whenever there is a dynamic loading, such as land vehicles, sea vessels and the aircrafts. The frequency interesting of the structural-acoustics systems range often from the “low-frequency” up to “high-frequency” regime, depends on the actual application. Different types of numerical simulation techniques have been developed for predictions, aiming to cover different frequency ranges. Most of these numerical prediction techniques can be categorized largely in two big groups: either deterministic or statistical methods. The former is suitable for lower frequency range, and the later for higher.

Finite element method (FEM) [1–3], a typical deterministic method, is mostly successfully used to model low frequency vibrational and acoustical behavior of structural-acoustic systems. However, the prediction for the frequency response of a complex structural-acoustic system becomes exponentially difficult as frequency increases. In practical level, two causes contribute to the difficulties in the higher frequency prediction. One main cause lies in the fact that a precise mathematical model can hardly be devised to capture the detailed deformation of the system, due to the decreasing wavelengths of deformation and the increasing model size. In the context of FEM, there is a “rule of the thumb” applied in controlling the elements sizes in order to obtain a relatively proper numerical solution [3]. Therefore, the computational effort of FEM will increase exponentially with the increasing of the frequency. The other cause is that the system response becomes more and more sensitive to small imperfections in the system with the increasing of the frequency, so that small manufacturing uncertainties may lead to huge variability in the frequency response. That means even a very detailed deterministic mathematical model cannot yield a reliable response prediction in a relatively higher frequency regime [4].

In recent years, the concern of developing a suitable estimation technique for structural-acoustic problems has shifted to the mid-frequency regime, where structural-acoustic systems exhibit the lot of

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http://dx.doi.org/10.1016/j.enganabound.2017.03.008
Received 11 September 2016; Received in revised form 23 January 2017; Accepted 4 March 2017
0955-7997/

uncertainties. Considering the difficulties involved in developing a reliable deterministic method for the prediction of higher frequency domain, lots of research efforts are contributed to develop an alternative that takes into account model uncertainties and variability. For dealing with the uncertainties involved in the structural-acoustics problems, probabilistic methods are most commonly chosen with acceptable efficiency, where the uncertainty is expressed using different type random variables [5]. The most robust and simplest probabilistic method is the well-known Monte Carlo simulation method (MCM). It often can offer the most direct and reliable result at the expense of the large computational load, and therefore MCM results are chosen as the reference result in this work [6,7]. In the context of finite element method, there are another two classical probabilistic variants: the spectral stochastic finite element method (SSFEM) [8] and stochastic perturbation finite element method approach (SPFEM) [9,10]. In SSFEM, several of random Hermite polynomials is used to approximate the response quantity of the systems [8]. In SPFEM, all the response vectors or matrixes related to the pre-defined random input variables are needed to be expanded based the Taylor series expansion [9,10]. All the quantities can be expanded at the first order or higher order approximations according to the Taylor series. In this work, for a low variability level of the design parameters, only the first order expansion is applied in the process without losing much accuracy, due to the its feasible strategy [11,12]. If only the first order approximations are applied, the relationship between the response and the random variables can be approximated as the linear functions. Therefore, the computational process can be largely simplified, and the expectations and the standard deviations of the response can be easily obtained through the traditional SPFEM. However, the calculation of the probability distributions of the response is commonly ignored, unless the random responses is able be approximated as Gaussian random fields [13]. In this work, several different random input variables with different distribution are defined, where the random responses is apparent non-Gaussian. In order to observe the distributions of the random response, a change-of-variable technique is introduced [14,15]. There are two important steps for the calculation of the probability density functions (PDF) and cumulative density functions (CDF): firstly, the first order Taylor expansion technique helps to produce the linear functions between the inputs and responses. Secondly, the change-of variable can help to produce the PDF and CDF of the responses. Thus, the combination of classical SPFEM and the change-of-variable technique provide a promising approach for the comprehensive analysis of uncertainties.

The deterministic method (FEM) is the key part for guaranteeing the accuracy in the probabilistic analysis. In the context of FEM, some more efficient techniques have been proposed to extend the frequency range of deterministic method(FEM) that are usual meant for low frequency problems. This category includes: stabilized methods [16,17],enriched methods [18,19], reduction techniques [20,21], higher order techniques [22,23],wave finite elements [24], and the ultra-weak variational formulation [25]. However, the use of the deterministic method (FEM)always entails some inherent drawbacks, which are closely associated with the well-known “overly-stiff” feature of FEM and its sensitivity to numerical pollution errors. Some researches [26–29] shows that the fundamental solution for eliminating the pollution errors is to soften the “stiffness” of the numerical structural-acoustic systems. Actually, a series of “soften” or “smoothing” gradient techniques have already been applied successfully into the traditional FEM frame [30–36], which are originally and extensively studied in meshfree methods [37,38] Smoothing gradient operations are performed on the newly constructed element integrate domains (smoothing domains), which can be built upon the nodes or edges of element. If the smoothing domain is constructed based on the nodes of elements, the node-based smoothed finite element method (NS-FEM) [30] can be built. However, research found that NS-FEM has overly soften the system stiffness, leading to an “overly-softy” feature. It is temporal instable in the calculation in the dynamic analysis [31]. Then, some researcher constructed the smoothing domain based on the edges of the elements and established a new method called ES-FEM [32–36]. In comparison with the traditional FEM or the above mentioned NS-FEM, the ES-FEM possesses suitable soften effects and help to build a reasonable system stiffness. And these effects enable the ES-FEM model to show neither “overly soft” nor “overly stiff” features. Therefore, the ES-FEM is able to reduce the desperation errors and thus enhance the accuracy of the prediction results. And the ES-FEM results are often found super convergence and ultra-accurate [32–36]. Given the superior performance of the edge-based smoothing technique (ES), it is natural to expect that the ES-FEM will greatly reduce the numerical dispersion error and obtain accurate results for acoustic problems. Thus, the embedding of the edge-based smoothing technique (ES) and change-of-variable technique into a similar hybrid stochastic perturbation FEM frame (SP-ES-FEM)presents a promising combination for solving structural-acoustic problem. The newly constructed ES-SP-FEM will be expected to efficiently and effectively evaluate the response probability density function and cumulative density functions. This is also the essential motivation of this work.

Based upon the above analysis, a stochastic perturbation ES-FEM(SP-ES-FEM), which combines the first order stochastic perturbation technique with the edge-based smoothed finite element approach is proposed for structural-acoustic coupling problems. The paper is organized as follows: in the Section 2, the basic principles of probability theory and the ES-FEM for 3D acoustics problem are briefly described, respectively. In the Section 3, the hybrid SP-ES-FEM equations are constructed in order to achieve a first order perturbation stochastic approach for structural-acoustic coupling problems problem. In Section 4, numerical examples and applications are presented to demonstrate the performance of the SP-ES-FEM for the frequency response analysis of structural-acoustic coupling problems. Finally, a summary is given in Section 4 to conclude this work.

2. Basic principles for deterministic structural-acoustic systems

Consider a structural-acoustic problem with thin elastic plate and an acoustic cavity, as shown in Fig. 1. The structural-acoustic system can be divided into a plate subsystem and an acoustic subsystem as shown in Fig. 1. Two subsystems are coupled exclusively through the interface Γ_{S_d} which is described by a set of degrees of freedom.

The vibrating structure is modeled using the ES-FEM method [37,38], where the 3-node linear triangle plate elements are based on the Reissner-Mindlin plate theory. For overcoming the shear locking problem existing in the Reissner-Mindlin plate theory, the well-known discrete shear gap (DSG) technique is applied [39-40]. The ES-FEM formulation for structural domain then can be written as:

$$\mathbf{M}_s\ddot{\mathbf{u}} + \overline{\mathbf{K}}_s\mathbf{u} = \mathbf{F}_s, \quad \text{in } \Omega_s \tag{1}$$

in which $\mathbf{u}$ is defined as the unknown field variable displacements, $\mathbf{M}_s,\overline{\mathbf{K}}_s$ is the lumped mass matrix and the edge-based smoothed

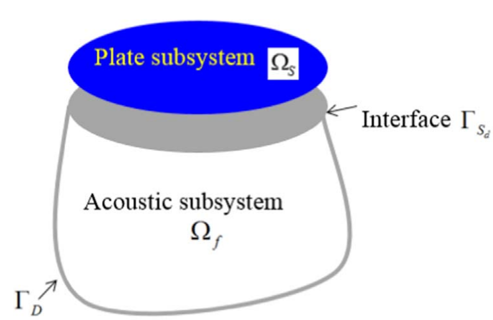

Fig. 1. Thestructural-acoustic coupling system.