Engineering Analysis with Boundary Elements 80 (2017) 152–161

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A boundary element and level set based bi-directional evolutionary structural optimisation with a volume constraint

Baseer Ullah[a,*], Jon Trevelyan[b], Siraj-ul-Islam[a]

[a] University of Engineering and Technology, Peshawar, Pakistan
[b] School of Engineering and Computing Sciences, Durham University, South Road, Durham DH1 3LE, UK

ARTICLE INFO

Keywords:
Topology optimisation
Boundary element method
Level set method
BESO

ABSTRACT

A new topology optimisation algorithm is implemented and presented for compliance minimisation of continuum structures using a volume preserving mechanism which effectively handles a volume constraint. The volume preserving mechanism is based on a unique combination of the level set method and a boundary element based bi-directional evolutionary structural optimisation approach using a bisectioning algorithm. The evolving structural geometry is implicitly represented with the level sets, efficiently handling complex topological shape changes, including holes merging with each other and with the boundary. Numerical results for two-dimensional linear elasticity problems suggest that the proposed adaptation provides smooth convergence of the objective function and a more robust, smoother geometrical description of the optimal design. Moreover, this new implementation allows for efficient material re-distribution within the design domain such that the objective function is minimised at constant volume. The proposed volume preserving mechanism can be easily extended to three-dimensional space.

1. Introduction

The main goal of structural optimisation is to provide an optimal design which should effectively comply to its intended objectives and at the same time satisfies the constraints imposed upon it. The demand for low-cost, light weight and high performance structures can be addressed through the development of high performance structural optimisation methods. Among the three types of structural optimisation, i.e. size, shape and topology, topology optimisation is the most beneficial from economic perspective and the most challenging from engineering perspective. According to [1], topology optimisation methods can be broadly classified into density based and level set based methods. In density based methods, the geometry of the structure is represented through a material distribution of two or more phases, e.g. [2,3], etc. In the second category an implicit boundary description is used to represent the structural geometry, e.g. [4–8], which are based on the level set method (LSM) [9].

In the LS based optimisation techniques, the performance of an evolving structural geometry can be evaluated using different geometry mapping approaches. According to [1], the most commonly used approaches are immersed boundary and conforming discretisation. There also exists another approach, where a fixed Eulerian mesh can be used for the LSM implementation and a body conforming approach for the evolution of the structural response. The body conforming approach can be based either on:

- the finite element method (FEM) based domain discretisation
- the boundary element method (BEM) based boundary only discretisation

The reduction of problem dimensionality with the use of BEM based body conforming mapping is very attractive as compared to the FEM based domain discretisation. In the literature of structural optimisation, researchers combined the BEM with the LSM for the solution of optimisation problems in both two and three-dimensions, e.g, compliance minimisation [10–13], sound scattering [14,15], heat conduction [16], etc.

An improvement in the structural performance of a candidate design can be based either on the shape sensitivity information (e.g. [6,17–19]) or through an evolutionary approach based on a criterion such as von Mises (e.g. [4,20]). The basic concept of evolutionary structural optimisation (ESO) is based on the progressive removal of inefficient materials, which evolves the structure towards an optimum [21,22]. The bi-directional evolutionary structural optimisation (BESO) presented in [23] also allows for efficient material to be added at the same time as the inefficient material is removed.

---

[*] Corresponding author.
E-mail address: baseerullah@gmail.com (B. Ullah).

http://dx.doi.org/10.1016/j.enganabound.2017.02.012
Received 20 July 2016; Received in revised form 4 December 2016; Accepted 10 February 2017

The removal and addition of materials in most of the finite element (FE) based BESO approaches are linked with the element removal and addition, which provides optimal designs with checkerboard patterns and jagged edges. Therefore, filtering process is always required to minimise the occurrence of these undesirable effects [24,25]. In the boundary element (BE) based BESO approaches [26,27] the material removal is accomplished through hole insertion and boundary movements, and addition through boundary movements only. However, due to the explicit geometry representation adopted in [26], special care is always required when hole merges with each other and with the boundary. The BE based BESO approach has been further integrated with the LSM for the solution of both two and three-dimensional optimisation problems in [12,28], which allows for the complex topological changes to take place automatically. As reported in the literature, the BE based BESO approaches largely eliminate the common problems occur in the FE based approaches, e.g. checkerboard patterns, jagged edges and mesh dependency. However, these methods are based on the target volume based stopping criterion instead of the most desirable, i.e. the minimisation of the objective function at constant volume criterion, which would provide optimal designs with improved performance.

A new optimisation method presented in this paper is based on a compliance minimisation objective function with a volume constraint, for linear elastic problems. In order to exactly satisfy the volume constraint, a novel methodology has been proposed for the constant volume preserving mechanism within the BEM and LSM framework. The proposed implementation exactly satisfies the volume constraint and, in addition, allows us to monitor the structural performance through a direct measurement of the compliance at constant volume during the optimisation process. The volume preserving mechanism is based on the bisectioning algorithm, which precisely adjusts the material removal in accordance with the material addition.

The effectiveness of the proposed implementation is thoroughly evaluated through the numerical examples presented and it has been observed that this new algorithm provides smooth convergence of the objective function and better geometrical description of the final design, i.e., the optimal geometries produced are smoother, and have more uniformly sized members, than those reported in [20,21,26,28]. In addition, this new implementation allows us to evaluate its intended purpose of minimising the objective function at constant volume, which is of a paramount importance for designing high performance structures. Therefore, this is a clear advantage of this method over the those presented in [20,26,28], where the optimal designs are based on the minimisation of the specific strain energy without incorporating the constant volume constraint. Hence, in each of the optimisation problem considered in this study, the performance of the proposed implementation is exceptional and the minimisation of compliance at constant volume has also been accomplished.

This paper is organised as follows. In Section 2, we discuss the BE and LS based BESO approach. In Section 2.5 the implementation details of the volume preserving algorithm are presented. The optimisation procedure is provided in Section 3. In Section 4, we present numerical examples, and discuss the performance of the proposed optimisation method. The paper closes with some concluding remarks in Section 5.

2. The BE and LS based BESO approach

A classical problem in structural optimisation is to find the stiffest structure with a given volume of the material. According to the BESO approach, a structure can be optimised through the progressive removal of inefficient and addition of efficient materials based on the sensitivity information. In the current implementation the design sensitivities are evaluated through the BEM and the LSM is then used to evolve the structural geometry in accordance with the BESO criterion. The integration of the various numerical techniques used in this study is discussed in detail as follows.

2.1. Problem statement

In the current implementation the design objective is to find the optimal topology of a structure with minimum compliance subject to a volume constraint. Consider an elastic structure with analysis domain Ω and boundary Γ. The boundary Γ is decomposed such that

$$\Gamma = \Gamma_0 \cup \Gamma_1 \cup \Gamma_2 \tag{1}$$

where Γ_0 corresponds to regions having Dirichlet boundary conditions (where displacements are zeros), Γ_1 corresponds to non-homogeneous Neumann boundary conditions (where tractions are prescribed) and Γ_2 corresponds to homogeneous Neumann boundary conditions (traction free). Γ_0 and Γ_1 are fixed and Γ_2 is allowed to vary during the optimisation process.

The optimisation problem can be expressed as finding Γ_2 to minimise the compliance (i.e. a measure of the strain energy), subject to the volume constraint. Mathematically, the optimisation problem can be stated as:

$$\begin{aligned} &\text{Minimise: } J(u) = \int_\Gamma \frac{1}{2} t_i u_i d\Gamma \\ &\text{Subject to: } G = \int_\Omega d\Omega - V_t = 0 \end{aligned} \tag{2}$$

where t_i and u_i are the traction and displacement in the direction i, and V_t is the target volume.

According to the BESO concept, the low sensitivity (stress or strain energy) regions within a structure reflect the inefficient material utilisation and the high sensitivity regions indicate insufficient material. Therefore, the progressive removal and addition of material in the BESO based optimisation method allows efficient material re-distribution for the prescribed volume of the material, accompanied with minimisation of the objective function. Hence, this provides optimum structure with near the same (safe) sensitivity (stress, strain energy) level.

2.2. Boundary element analysis

The BEM is used as a structural analysis tool in the current implementation. Due to the boundary only dicretisation the structural response can be directly evaluated at the nodal points associated with the elements. Moreover, in a BE analysis stresses (or any other required property) inside the design domain can be calculated at internal points as a post processing step. The current implementation uses the boundary element analysis software Concept Analyst (CA) [30]. Therefore, the complete optimisation code is fully integrated within the CA.

2.3. Design sensitivity analysis

In most of the FE based BESO approaches the removal and addition of materials is linked with the element removal and addition, which provides optimal designs with checkerboard patterns and jagged edges. Therefore, additional measures are always adopted to minimise the occurrence of these undesirable effects. However, in the BE based BESO approaches [26,28] the material removal is accomplished through hole insertion and boundary movements, and addition through boundary movements only, without any undesirable effects. The topological and shape sensitivities have been used to identify regions with in the structure to be modified accordingly.

In the current implementation, both these sensitivities are based on the von Mises stress criterion, which drive the removal and addition process in order to achieve a minimum of the objective function. According to the comparative study presented in [30,31], the criterion of von Mises stress in the classical ESO method is equivalent