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Dual reciprocity boundary node method for convection-diffusion problems

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ABSTRACT

This paper presents a dual reciprocity boundary node method(DHBNM) for steady-state convection-diffusion problems with variable velocities. In this formulation, the solutions are composed of a particular and a complementary solutions, where the former is solved by boundary node method, and the latter is obtained by dual reciprocity method(DRM). The velocity field is decomposed into an average and a perturbation ones, which is treated using DRM by some steps of approximating process. A reverse procedure for the particular solution is developed to overcome the difficulty of getting the analytical solution of the particular solution. Numerical studies have validated that the DHBNM can produce high accuracy and stability.

1. Introduction

A convection-diffusion problem is a combination of convection and diffusion, which describes a physical phenomena where particles, energy, or some other physical quantities are transferred inside a physical system. For example, heat and mass transfer coupled with fluid flows could be mathematically described by convection-diffusion equations. Unfortunately, using analytical methods to obtain the exact solutions of the partial differential equations is a challenging task, thereby many numerical methods have been developed for solving this type of problems. Although a great success of applying the finite volume method (FVM) [1,2] and finite element method(FEM) [3,4] has been achieved, there are still some drawbacks. For example, FVM and FEM strongly depend on mesh properties, and both are time-consuming, especially for 3D problems. Thus, meshless methods have been developed recently to overcome those difficulties, which have received greater attention and attracted significant applications in the past decades.

In the past years, a lot of meshless methods have been adopted to solve heat transfer and fluid dynamics problems, e.g. Cleary and Monaghan [5] firstly applied the smooth particle hydrodynamics(SPH) method to solve unsteady-state heat conduction, which was later employed widely for the incompressible flow problems [6] and free surface flow problems [7]. As a widely used meshfree method, the element free Galerkin (EFG) method was also applied to solve 2D and 3D heat conduction problems [8]. Fries and Matthies [9] coupled meshfree method EFG and meshbased FEM for the incompressible Navier-Stokes equations. Lin and Atluri [10,11] developed some unwind schemes for the meshless local Petrov-Galerkin (MLPG) method to compute convection-diffusion problems and incompressible Navier-Stokes equation, and then Liu and Tan [12] applied it for solving coupled radiative and conductive heat transfer. Wu et al. [13] used MLPG for two-dimensional heat conduction problems.

Besides, the finite point method were applied for convective transport and fluid flow problems by Onate et al. [14]. Gunther et al. [15] employed the reproducing kernel particle method (RKPM) to solve viscous, compressible flow problems. Liu et al. [16] proposed a meshfree weak-strong (MWS) form method and applied it to solve the incompressible flow problems. Liu et al. [17] proposed a meshless weighted least-squares (MWLS) method to compute steady and unsteady-state heat conduction problems. Shu et al. [18] developed the local radial basis function-based differential quadrature (RBF-DQ) method to solve two-dimensional incompressible Navier-Stokes equations. Wu et al. [19,20] applied the node-based smoothed point interpolation method (NS-PIM) to solve 3D heat transfer problems and the substrate temperature in plasma spraying process. Cockburn and Shu [21] proposed the local discontinuous Galerkin method for time-dependent convection-diffusion problem. Yang and Zhu [22] applied it then for 2D singularly perturbed convection-diffusion problems.

As a boundary type meshless method, the boundary node method (BNM) was firstly proposed by Mukherjee [23], in which moving least square (MLS) was applied to boundary integration equations. Although no element is needed for the variable interpolation for BNM, background elements are required for ‘energy’ integration. In order to address these issues, Zhang and Yao [24] applied hybrid displacement

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variational formulation to MLS, and proposed the hybrid boundary node method (HBNM). Later, based on HBNM and rigid body displacement, Miao and Wang [25] proposed a meshless method of singular hybrid boundary node method (SHBNM). Besides, Gu et al. [26,27] proposed a meshless singular boundary method for three-dimensional elasticity. Furthermore, to solve the complicated problems of domain integration encountered in meshfree methods, Racz and Bui [28] developed two adaptive numerical integration approaches based on mapping techniques. Furthermore, to avoid domain integral from the inhomogeneous term of the governing equation, Yan and Wang [29,30] employed dual reciprocity method(DRM) [31,32] to HBNM, and proposed the dual reciprocity hybrid boundary node method (DHBNM) to solve inhomogeneous, dynamic, nonlinear problems etc. Later, based on Shepard interpolation method and Taylor expansion, Yan and Feng [33] proposed a new shape function constructing method, i.e., the Shepard and Taylor interpolation method(STIM), and further proposed a new meshless method of dual reciprocity hybrid boundary node method based on STIM.

According to those literature review, DHBNM can be applied for inhomogeneous, dynamic, nonlinear problems, and has high accuracy and stability. It has been claimed that the boundary element method (BEM) or some boundary type meshless method perform better than the FEM and FVM in solving the convection-diffusion problems due to the fact that the convection terms have been inherently included into the fundamental solution for the convection-diffusion operator. However, convection-diffusion problems is complicate and nonlinear, and the traditional DHBNM cannot easily solve those problems. Therefore, in this paper, a new meshless method DHBNM is proposed for convection-diffusion problems. Firstly, the velocity field of convection-diffusion problem is decomposed into an average and a perturbation parts, the former can be treated by HBNM via the fundamental solution, and the latter can be treated using DRM by some steps of approximate process. The solution of convection-diffusion equation is divided into the a particular and a complementary solutions, in which the complementary solution is solved by boundary node method, and the particular solutions are obtained by DRM. Unfortunately, the particular solution cannot be obtained analytically. Thereby a reverse procedure for the particular solution is developed to overcome the difficulty. Based on those theories, a new DHBNM is proposed to solve convection-diffusion problems, which is validated by numerical studies on its accuracy and stability.

2. Dual reciprocity boundary node method

Convection-diffusion problems by dual reciprocity boundary node method can be solved by a two-step numerical scheme. Firstly, DRM and radial basis function (RBF) interpolation are employed to evaluate the particular solution of the problem, and secondly, the homogeneous solution is calculated by using hybrid boundary node method.

2.1. Governing equation

Consider the following two-dimensional steady-state convection-diffusion equation including first-order reaction, which can be written as [34,35].

$$D\nabla^2 u - v_x\frac{\partial u}{\partial x} - v_y\frac{\partial u}{\partial y} - ku = b(x, y) \tag{1}$$

$$u = \hat{u}(x, y), \text{ on } S_u \tag{2a}$$

$$q = \frac{\partial u}{\partial n} = \hat{q}(x, y), \text{ on } S_q \tag{2b}$$

in which $v_x = v_x(x, y)$ and $v_y = v_y(x, y)$ are the components of the velocity vector $\mathbf{v} = (v_x, v_y)$; D is the diffusivity coefficient, of which the medium is assumed homogeneous and isotropic in this paper; k represents the reaction coefficient; u can be referred as a potential, which can be interpreted as temperature for heat transfer problems, concentration for dispersion problems, etc.; q is the normal flux;$\hat{u}(x, y)$ is the boundary value on Dirichlet boundary S_u, and $\hat{q}(x, y)$ is the boundary value on Neumann boundary S_q; $b(x, y)$ is the inhomogeneous term of the equation.

While the fundamental solutions are only available for the case of constant velocity field, while $v_x = v_x(x, y)$ and $v_y = v_y(x, y)$ are generally not constant. Thus, the variable velocity components $v_x(x, y)$ and $v_y(x, y)$ are decomposed into average terms $\bar{v}_x$ and $\bar{v}_y$ and perturbations $P_x = P_x(x, y)$ and $P_y = P_y(x, y)$. Thus,

$$v_x(x, y) = \bar{v}_x + P_x(x, y) \tag{3a}$$

$$v_y(x, y) = \bar{v}_y + P_y(x, y) \tag{3b}$$

in which $\bar{v}_x$ and $\bar{v}_y$ are constant. Eq. (1) can then be rewritten as

$$D\nabla^2 u - \bar{v}_x\frac{\partial u}{\partial x} - \bar{v}_y\frac{\partial u}{\partial y} - ku = b(x, y) + P_x\frac{\partial u}{\partial x} + P_y\frac{\partial u}{\partial y} \tag{4}$$

According to DRM formulation, the solution of Eq. (4) can be divided the particular solution and the complementary solution. The complementary solution can be solved by boundary node method, and the particular solution is obtained by DRM. Therefore the solution of u can be expressed as

$$u = u_c + u_p \tag{5}$$

in which u_c and u_p are the complementary solution and the particular solution of the problem, respectively.

The particular solution u_p satisfies the following equation in the whole space,

$$D\nabla^2 u_p - \bar{v}_x\frac{\partial u_p}{\partial x} - \bar{v}_y\frac{\partial u_p}{\partial y} - ku_p = b(x, y) + P_x\frac{\partial u}{\partial x} + P_y\frac{\partial u}{\partial y} \tag{6}$$

and it does not necessarily satisfy the boundary conditions. Thereby the complementary solution u_c should satisfy the homogeneous equation and the modified boundary conditions, which can be written as

$$D\nabla^2 u_c - \bar{v}_x\frac{\partial u_c}{\partial x} - \bar{v}_y\frac{\partial u_c}{\partial y} - ku_c = 0 \tag{7}$$

$$u_c = \hat{u}(x, y) - u_p, \text{ on } S_u \tag{8a}$$

$$q_c = \hat{q}(x, y) - \frac{\partial u_p}{\partial n}, \text{ on } S_q \tag{8b}$$

The following Section presents the computing formulation for the complementary solution and the particular solution.

2.2. Dual reciprocity approach

Based on the DRM [29,30], the inhomogeneous term $b(x, y) + P_x\frac{\partial u}{\partial x} + P_y\frac{\partial u}{\partial y}$ of Eq. (6) can be approximated by

$$b(x, y) + P_x\frac{\partial u}{\partial x} + P_y\frac{\partial u}{\partial y} \approx \sum_{k=1}^{L} f(r_k)\alpha_k \tag{9}$$

in which α_k are the unknown interpolating coefficient, and it is determined by the inhomogeneous term and the basis function; L is the total interpolating nodes on the domain and its boundary; r_k denotes the Euclidean distance between each calculating node (x, y) and the source node (x_k, y_k); and $f(r_k)$ is the basis function of DRM.

If a basic form of particular solution $\bar{u}$ is defined, it should satisfy that

$$D\nabla^2\bar{u} - \bar{v}_x\frac{\partial\bar{u}}{\partial x} - \bar{v}_y\frac{\partial\bar{u}}{\partial y} - k\bar{u} = f(r) \tag{10}$$

In this case, one can easily get the particular solution via the coefficient α_k and the basic form of particular solution $\bar{u}$, which can be expressed as