Engineering Analysis with Boundary Elements 76 (2017) 10–25

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Three-dimensional dynamic ring load and point load Green's functions for continuously inhomogeneous viscoelastic transversely isotropic half-space

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ARTICLE INFO

Keywords:
Green's function
Ring load
Point load
Dynamic response
Transversely isotropic
Continuously inhomogeneous
Viscoelastic

ABSTRACT

An analytical formulation is presented for three-dimensional Green's functions of continuously inhomogeneous linear viscoelastic transversely isotropic half-space subjected to either ring load or point load. It is assumed that the elastic moduli of the half-space vary in terms of depth as bounded exponentially functions, while the mass density is constant. The method of potential functions is used to partially decouple the governing equations, after which Fourier series expansion followed by Hankel integral transforms is applied to transform the partial differential equations to ordinary differential equations (ODEs) with variable coefficients. Then, Frobenius series method is employed to determine the potential functions and then the displacements and stresses in the transformed domain, which are used to evaluate these functions in physical domain. The validity of the formulations and numerical process is shown for several simplified cases comparing with the known solutions in the literature. Finally, the displacement and stress Green's functions are presented for several physical cases due to either unit ring load or unit point load. The results show that if the shear waves are produced in the interested direction, both inhomogeneity parameters and material damping may change the dynamic response of the half-space significantly, especially in high frequencies.

1. Introduction

Green's functions are the heart of many analytical and numerical techniques employed in solving numerous problems in the mechanics and physics of solids. These functions can be used as the fundamental solutions in boundary element method [1,2], which is a powerful numerical/analytical method for solving some specific boundary value problems, especially in the subject of soil-structure interaction. Green's functions are also employed in several other engineering and physical problems such as material characterization, damage/inclusion detection, wave propagation, seismology [3], inverse problems and impedance functions of foundations supporting structures or industrial machineries [4]. Several principle parameters may affect the Green's function of the medium including material properties (anisotropy, viscoelasticity, inhomogeneity), the configuration of the interested domain (full-space, half-space, layered domain, domain including inclusions, etc.), and the load specifications. Reviewing the investigations exist in the literature about material properties of the soil medium reveals some main characteristics. In-situ measurements such as cross-hole tests for isotropic materials indicate that the wave velocity in the soil medium varies with depth [5], which is mainly due to change of the elastic moduli, and show that even un-layered soil medium is inhomogeneous in depth. On the other hand, elastic moduli of the soil medium in horizontal and vertical planes are different, and its behavior may be modeled as transversely isotropic behavior [6]. Moreover, studies done by different researchers such as Zhang et al. [7] show that the soil half-space is a viscoelastic medium with energy dissipation property, which can be represented by damping ratio. In the other words, a realistic soil has three main characteristics as inhomogeneity, transversely isotropy, and viscoelasticity. These features affect both the wave propagation in the media and dynamic response of the foundations resting on the soil.

Several researches can be found in the literature studying these characteristics and their effects on the dynamic response of the medium with some simplified assumptions, most of them without considering all of these characteristics simultaneously. This paper concerns with these properties of the soil simultaneously, and a mathematical solution accompanied with a numerical procedure is developed to derive the displacement and stress Green's functions for a continuously inhomogeneous viscoelastic transversely isotropic half-space.

A summary of researches in the fields interested in this paper has been presented by Cheshmehkani and Eskandari-Ghadi [8], and main features of wave propagation and dynamic response of continuously

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http://dx.doi.org/10.1016/j.enganabound.2016.12.009
Received 19 October 2016; Received in revised form 1 December 2016; Accepted 22 December 2016

Nomenclature

A_i $(i = 1\sim6)$ Coefficients determined by boundary and regularity conditions
a Radius of the ring load
a_n, b_n, d_n Coefficients of different terms of power series
C_{0ij} Elastic moduli at free surface z=0
$C_{\infty ij}$ Elastic moduli at infinite depth
$C_{0ij}/C_{\infty ij}$ Inhomogeneity parameter
E , E_0 Young's moduli in the plane of transverse isotropy in general and at free surface z=0
E' , E'_0 Young's moduli in the direction normal to the plane of transverse isotropy in general and at free surface z=0
$\mathbf{f}_h(r, \theta, z, t)$ Horizontal component of the ring load
F_h Magnitude of the ring load in horizontal direction
$\mathbf{f}_v(r, \theta, z, t)$ Vertical component of the ring load
F_v Magnitude of the ring load in vertical direction
G , G_0 Shear modulus in the plane normal to the axis of symmetry in general and at free surface $z = 0$
G', G'_0 Shear modulus in planes normal to the plane of transverse isotropy in general and at free surface $z = 0$
J_m Bessel function of the first kind and m^{th} order
L Unit length for point load case or a (radius) for ring load case
P, Q, R Components of applied time-harmonic surface load in $r-$, $\theta-$ and $z-$ directions
r, θ, z Radial, angular and vertical coordinates in cylindrical coordinat system
t Time variable
u_i Displacement component in i direction $(i = r, \theta, z \ \ or \ \ i = x, y, z)$
$\hat{u}_i^{\,k}$ Displacement response in $i-$ direction due to unit point/ring load in $k-$ direction
x, y, z Coordinates in Cartezian coordinate system
β Inhomogeneity parameter
$\delta(r)$ Dirac-delta function
$\varepsilon_{ij}(i, j = \{r, \theta, z\} \ or \ \{x, y, z\})$ strain tensor
η Transformed depth variable
ρ Material density
$\sigma_{ij}(i, j = \{r, \theta, z\} \ or \ \{x, y, z\})$ Stress tensor
$\hat{\sigma}_{ij}^{\,k}$ Stress response σ_{ij} due to unit point/ring load in $k-$ direction
υ Poisson's ratio, $\upsilon = -\ \varepsilon_{\theta\theta}/\varepsilon_{rr}$ when subjected to the stress σ_{rr} or $\upsilon = -\ \varepsilon_{rr}/\varepsilon_{\theta\theta}$ when subjected to the stress $\sigma_{\theta\theta}$
υ' Poisson's ratio, $\upsilon' = -\ \varepsilon_{rr}/\varepsilon_{zz} = -\ \varepsilon_{\theta\theta}/\varepsilon_{zz}$ when subjected to the stress σ_{zz}
ω_0 Non-dimensional frequency
ω Angular frequency
ξ Hankel's integral transform parameter
ζ Material damping ratio

nonhomogeneous isotropic half-space comparing with the homogeneous one are described there based on these studies. Most of the studies done for inhomogeneous media are in the context of isotropic materials, while the study of wave propagation and dynamic responses in continuously inhomogeneous transversely isotropic media are very limited in the literature, mainly due to mathematical complexities and the difficulties in numerical evaluations. Main studies in this field include Wang et al. [9,10], and Eskandari-Ghadi and Amiri-Hezaveh [11], who all considered similar distributions for elastic modulus and the density in depth of transversely isotropic half-space, which results in a constant wave velocity in depth. Moreover, both elastic moduli and density in these studies approach to either a very large value or a very small value at a large distance from a reference point. These researches, although show the mathematical difficulties concerning continuously inhomogeneous transversely isotropic materials, they do not consider a general case in continuously inhomogeneous media.

Cheshmehkani and Eskandari-Ghadi [8] have studied the dynamic response of continuously nonhomogeneous transversely isotropic viscoelastic half-space assuming a bounded distribution of elastic moduli as $C_{ij}(z) = C_{\infty ij} - (C_{\infty ij} - C_{0ij})e^{-\beta z}$ accompanied with a constant density, subjected to an axisymmetric vertical load such as point load or distributed load applied on a circular patch. Without being needed any potential function, they employed Hankel integral transforms accompanied with Frobenius series method to solve the boundary value problem, and they indicated that when vertical load is applied, inhomogeneity parameters affect the dynamic response of the half-space considerably, especially at the vicinity of the free-surface.

In this paper, the Green's functions of continuously nonhomogeneous transversely isotropic half-space are presented for an arbitrary surface ring load of unit magnitude and also its degeneration to unit point load. The elastic moduli in this study vary in depth with a bounded distribution within two limits denoted as C_{0ij} and $C_{\infty ij}$, while the mass density is constant. These assumptions for elastic moduli and mass density have already been investigated by Vrettos and Prange [12] who experimentally studied the soil profile of the uniform sand deposit, and evaluated the appropriate inhomogeneity parameters for it, assuming the bounded distribution for shear modulus and constant density. The resulted inhomogeneity parameters have been used in some other studies of Vrettos such as [13–17]. Here, viscoelastic behavior is also considered to model the energy dissipation of the soil [7]. In fact, the solution presented in the previous paper of the authors [8], which was for axisymmetric vertical load is extended in this paper to 3D load cases with the use of appropriate new potential functions to partially decouple the governing equations of motion, and then Fourier series expansion followed by Hankel integral transforms is employed. In this way, the coupled governing differential equations are transformed to some ordinary differential equations with variable coefficients, which are solved using Frobenius series method in the next step. The unknown constants appeared in the integration process are specified by satisfying boundary conditions at the free surface and radiation condition at infinity. Employing this procedure, at first the potential functions and then the displacements and the stresses are determined in Hankel/Fourier transformed domain. Afterwards, the theorem of inverse Hankel integral transforms followed by Fourier series expansion are applied to specify the displacement and stress responses in the real domain. Eventually, the components of displacement- and stress-Green's functions are numerically evaluated for some material specifications and for several excitation frequencies to investigate the effect of inhomogeneity in different frequencies. The results show that the dynamic responses of inhomogeneous transversely isotropic media mainly differ from the homogeneous one, especially when shear waves are produced due to the load. The formulations of this paper can be used as the kernel in integral base numerical methods such as boundary element method.

2. Statement of the problem

The domain of the problem considered in this paper is a viscoelastic transversely isotropic half-space, where the elastic moduli vary in depth with a bounded exponential function within two limits. The axis of symmetry of the medium is normal to the horizontal surface and a cylindrical coordinate system (r, θ, z) is attached on the free surface as reference in such a way that its $z-$ axis is taken to be depth-wise and parallel to the axis of material symmetry. An arbitrary time-harmonic