Engineering Analysis with Boundary Elements 76 (2017) 59–68

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Why dual boundary element method is necessary?

Jeng-Tzong Chen[a,b,*], Ching-Yun Yueh[a], Yu-Lung Chang[a], Chun-Chiang Wen[a]

[a] Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung 20224, Taiwan
[b] Department of Mechanical and Mechatronic Engineering, National Taiwan Ocean University, Keelung 20224, Taiwan

A R T I C L E I N F O

Keywords:
Dual boundary element method
Degenerate boundary
Breakwater
Hypersingular integral equation
Zero-thickness

A B S T R A C T

Dual boundary integral equation (BIE) was developed for problems containing degenerate boundaries in 1988 by Hong and Chen [Journal of Engineering Mechanics-ASCE, **114**, 6, 1988] and was termed the dual boundary element method (BEM) in 1992 by Portela et al. [International Journal for Numerical Methods in Engineering, **33**, 6, 1992]. After near 30 years, the dual BIE/BEM for the problem containing a zero-thickness barrier was revisited mathematically to study the rank deficiency from the viewpoint of the updating term and the updating document of singular value decomposition (SVD) [Journal of Mechanics, **31**, 5, 2015]. In this paper, we revisit the dual BEM from the physical point of view. Although there is no zero-thickness barrier in the real world, it is always required to simulate a finite-thickness degenerate boundary to be zero-thickness in comparison with sea, air or earth scale. For example, a sheet pile, a screen, a crack problem, a thin airfoil and a breakwater were modeled by the geometry of zero-thickness. The role of the dual BEM is evident since Lafe et al. [Journal of the Hydraulics Division-ASCE, **106**, 6, 1980] used the conventional BEM to model the finite-thickness pile wall to geometrically approximate zero-thickness barrier but numerically yielding divergent solution. On the contrary, we physically model the finite-thickness breakwater as a zero-thickness barrier. The breakwater is employed as an illustrative case to demonstrate that the dual BEM simulated by a zero-thickness barrier can yield more acceptable results to match the experiment data in comparison with those of the finite thickness using the conventional BEM. Finally, a single horizontal plate and two dual horizontal plates in vertical direction and in horizontal direction are three illustrative cases to tell you why the dual BEM is necessary not only in mathematics but also in physics.

1. Introduction

The boundary element method (BEM) or so-called boundary integral equation method (BIEM) is an efficient tool for the engineering analysis. The advantage of the BEM/BIEM is the reduction of the mesh generation because the dimension of the problem is diminished by using the Green's third identity or Betti's law as well as Somigliana's identity. Over the past 30 years, applications of the BEM/BIEM have been found in many fields [1–6]. However, the conventional BEM/BIEM has four degenerate problems due to the singular matrix or ill-posed model, such as the degenerate scale [7,8], the fictitious frequency [9,10], the spurious eigenvalue [11,12] and the degenerate boundary [13]. It is found that the dual BEM plays an important role to solve the above rank-deficiency problems. Mathematically speaking, we can not ensure the uniqueness of the solution in case of these ill-posed models [14]. Dual BIEM as an elegant method to deal with rank-deficiency problems of the conventional BEM/BIEM was developed by Hong and Chen [15]. By using the dual BEM, the degenerate boundary problems, e.g., Darcy flow around the cutoff wall [15], a screen in an acoustic cavity [16,17] and a crack in an elastic body [18] were efficiently solved free of subdomain. The key concept of the dual BEM is the application of the hypersingular integral equation which provides sufficient constraints for unknown coefficients. The integral named "Hadamard principle value (HPV) [19]" (solid mechanics) or "Mangler's principle value [20]" (aerodynamics) is embedded in the hypersingular integral equation. In comparison with the singular integral equations, Hong and Chen [15] termed it "Dual BIE" and applied it to deal with degenerate-boundary problems. In 1989, the numerical implementation of dual BIEM was done for a cutoff wall in BEM meeting. In addition to the application of the hypersingular integral equation in mechanics by Martin et al. [21], Chen and Hong [22] presented a review article on the dual BEM. Recently, Chen et al. [23] revisited the dual BEM from the mathematical point of view by using the updating term and the updating document of the singular value decomposition (SVD). Both the right and left singular vectors of the SVD for the four influence matrices in the dual BEM were also discussed. The role of the

* Corresponding author at: Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung 20224, Taiwan.
E-mail address: b0209@mail.ntou.edu.tw (J.-T. Chen).

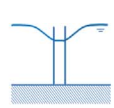
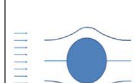
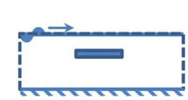
http://dx.doi.org/10.1016/j.enganabound.2016.11.011
Received 15 July 2016; Received in revised form 1 November 2016; Accepted 30 November 2016

hypersingular integral equation to provide full rank matrix is obvious after using the SVD updating technique from the mathematical point of view.

In the coastal engineering, the breakwater is constructed to protect the shoreline, marinas and coastal structures. It can also provide calm water areas for loading and unloading ships in the harbor. In 1950, Heins [24] was the first researcher to investigate the hydrodynamic performance of water waves over a semi-infinite submerged horizontal plane barrier in finite depths. Patarapanich et al. [25] measured the experimental data and used the finite element mzethod to discuss the dependence of the reflection coefficients on the length of the plate. Kojima et al. [26] conducted experiments to investigate the decomposition of regular and irregular waves by using a fixed submerged horizontal plate and a permeable vertical wall. One of the primary conclusions obtained is that a submerged horizontal plate may serve as an effective interception device against long waves. Based on the Galerkin approximation, Porter and Evans [27] accurately solved the scattering problem of waves by vertical barriers in infinite-depth and finite-depth water containing square-root singularities at the end of the barrier. There is a similar problem about the stress analysis for the crack problem in the reference [15]. In the solid mechanics, crack tip behavior dominates the crack growth. The asymptotic behavior at the tip relates to the stress intensity factor on the fracture and fatigue. In case of flow over a right-angled wedge, the singularity for the flow velocity in the bottom point of the right-angled wedge occurs. Similarly, the singularity also exists in the outflow tip for the saltwater intrusion problem [28]. Therefore, an enriched singular element to capture $\sqrt{r}$ behavior has been employed. Liu et al. [29] investigates the hydrodynamic performance of a submerged breakwater of two layer horizontal plates with a non-zero thickness. In the context of linear potential theory, an analytical solution for interaction of water waves with the plates is obtained by using the matched eigenfunction expansion method. Results show that the space between two plates affects slightly the reflection and transmission coefficients. Jin et al. [30] examined the hydrodynamic performance of the horizontal plate breakwater using the numerical wave flume based on the OpenFOAM. The viscous effect of fluid has been considered to obtain the flow characteristic and the energy dissipating mechanism of the submerged horizontal plate. Hsieh [31] and Wu [32] investigated experimentally the wave reflection and pressure forces by the wave over a series of submerged horizontal plates. Experimental results showed that the reflection coefficient was being affected with the plate length and submerged depth.

For the linking between mathematics and physics, some simplifications due to the scale contrast are convenient and useful for the modeling as shown in Table 1. For example, Dirac delta function can be considered as a lumped force applied in a very small area. In the text book of mechanics of material, we always model a wheel loading as a concentrated load [33]. Similarly, pumping in a well can be approximated as a point source/sink in the groundwater drawdown problem [34]. Furthermore, a local slippage of a geological fault or a defect in crystal structure can be considered as a displacement discontinuity or dislocation [35]. In the modeling, there is a simple case that a dipole can be used to simulate as an obstacle of a circular cylinder in the potential flow [36]. In aerospace engineering, von Karman [37] dealt with the airfoil problem by putting the source point on the mean curve of the airfoil. Recently, Young et al. [38] and Young et al. [39] also solved airfoil problems (NACA 2418 airfoil and NACA 0012 airfoil) of 2D potential flow by using the method of fundamental solutions (MFS) where sources are located on the Chord line of NACA 2418 airfoil and NACA 0012 airfoil, respectively. It indicates that the airfoil can be modeled by a mean line or a mean curve of the airfoil. Table 1 summarizes the simplification for the several physical problems in the real world. Therefore, choosing the suitable simplification in the model plays an important role in modeling the physical problem. As the uniform distributed force presented by the lumped force in mechanics, the submerged plate-type breakwater can be considered as a thin barrier with the impenetrable boundary condition below the water surface. For the simplification, the breakwater can be seen as a zero-thickness one causing the degenerate-boundary problem which can not be solved by using the conventional BEM. Lafe et al. [40] found that the geometry scheme of limiting to a zero thickness of the cut-off wall in the sheet-pile problem for the BEM can not numerically lead to the convergent solution to an exact solution with deviation as shown in Fig. 1. The BEM with the multi-domain discretization by adding the artificial boundary was used to deal with the degenerate-boundary problem, e.g., cutoff wall [40], crack problems [41], screen acoustics and thin barrier [42,43]. As mentioned earlier, Hong and Chen [15] solved it by using the dual BEM instead of the multi-domain BEM. Yueh and Tsaur [44] also employed the dual BEM to solve the problem of normal incident water wave past a submerged barrier. Chen et al. [45] used the dual BEM to deal with the problem of the oblique incident wave passing a thin submerged breakwater descending from the water surface to a depth. Then, the problem of incident wave passing a vertical thin submerged breakwater fixed on the bed is solved by using the dual BEM [46]. Unfortunately, the difference between the conventional BEM and dual BEM by comparing with the experimental data was not mentioned.

Table 1
The simplification of physical problems in the real world.

	Mechanics [31]	Groundwater [32]	Crystal [33]	Uniform flow [34]	Aerodynamics [35, 36, 37]	Breakwater (present study)
Real world						
Simplification