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Numerical simulation of three-dimensional double-diffusive natural convection in porous media by boundary element method

J. Kramer Stajnko*, J. Ravnik, R. Jecl

Faculty of Civil Engineering, Transportation Engineering and Architecture, Faculty of Mechanical Engineering, University of Maribor, Smetanova 17, SI-2000 Maribor, Slovenia

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ABSTRACT

The paper presents numerical results for three-dimensional double-diffusive natural convection in a cubic enclosure fully filled with fluid saturated porous media. Two opposite vertical walls of the enclosure are subjected to different values of temperature and concentration, which causes buoyant and diffusive flow in the porous media domain. Mathematical model is based on the Brinkman-extended Darcy formulation as a governing momentum equation, which is coupled with the energy and species equations. The three-dimensional boundary element method based solver was used to solve the obtained set of partial differential equations. The existing numerical algorithm primarily derived for the pure fluid flow simulations was adopted to simulate transport phenomena in porous media. It is based on the combination of single and subdomain boundary element method, which solves the velocity-vorticity formulation of the governing equations. In the paper the influence of some governing parameters, specially the Rayleigh number, Darcy number and buoyancy coefficient are investigated in order to analyze the heat and mass transfer through porous enclosure. The numerical code is verified by comparison of the results with available previous numerical data found in the literature.

1. Introduction

Problems of double-diffusive natural convection in porous media occur in many natural and engineering applications and have been recently intensively investigated. Enhanced attention has been dedicated to several environmental problems, e.g. transport of contaminant through water saturated soil in field of protection of groundwater resources and nuclear waste disposals. In such problems complex flow patterns are a result of combined actions of temperature and concentration gradients and the presence of porous media. There are many published analytical, numerical as well as experimental studies considering simultaneous heat and solute transfer in several different configurations of porous media domains. First, most commonly studied situation is the horizontal porous layer subjected to vertical temperature and concentration gradients. The critical conditions for the onset of convective motion were usually obtained on the basis of linear stability analysis. Murray and Chen [1] published a study where double-diffusive natural convection in horizontal porous layer is investigated experimentally. They reported about dramatic behavior of flow patterns in case when performing the double-diffusive experiments, with occurrence of three-dimensional cells. Trevisan and Bejan [2] studied the critical conditions for the onset of convective motion in an infinite horizontal porous layer. Rosenberg and Spera [3] published a study with numerical results of convection in porous medium heated from below with two opposing sources of buoyancy. They studied in detail the effect of Rayleigh number, Lewis number and buoyancy coefficient on the overall heat and mass transfer. Amahmid et al. [4] studied double-diffusive parallel flow analytically and numerically, where the analytical solution is based on the parallel flow approximation. The critical Rayleigh number for the onset of parallel flow is determined analytically as a function of Lewis number, buoyancy coefficient and Darcy number. Mahidjiba et al. [5] published numerical study of double-diffusive natural convection in a horizontal porous cavity using the linear stability analysis.

The second common configuration is the vertical cavity fully filled with saturated porous media where the vertical walls are maintained at horizontal temperature and concentration gradients. Two different types of phenomena which were studied can be found, namely the resulting thermal and solutal buoyancy forces can have aiding or opposing influence on each other. Trevisan and Bejan [6,7,2] published comprehensive analytical and numerical studies considering double-diffusive natural convection in a porous enclosure where the vertical walls are maintained at different temperature and concentration values or uniform heat and mass fluxes. The developed analytical solutions are

* Corresponding author.
E-mail addresses: janja.kramer@um.si (J.K. Stajnko), jure.ravnik@um.si (J. Ravnik), renata.jecl@um.si (R. Jecl).

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Nomenclature

α	effective thermal diffusivity of porous medium, $\alpha_e = \lambda_e / c_f$
β_C	volumetric expansion coefficient due to chemical species
β_T	volumetric thermal expansion coefficient
Γ	boundary of the computational domain
Λ	viscosity ratio, $\Lambda = \mu_{eff}/\mu$
λ_e	effective thermal conductivity of fluid saturated porous medium, $\lambda_e = \phi\lambda_f + (1-\phi)\lambda_s$
λ_f	thermal conductivity of fluid
λ_s	thermal conductivity of solid phase
μ	fluid dynamic viscosity
μ_{eff}	effective viscosity
ν	kinematic viscosity of the fluid
Ω	computational domain
ϕ	porosity
ρ	density
σ	heat capacity ratio
Θ	inner angle
$u^\star$	fundamental solution of the Laplace equation
$\vec{\omega}$	vorticity vector
$\vec{\xi}$	source or collocation point
$\vec{g}$	acceleration due to gravity
$\vec{n}$	unit normal vector
$\vec{r}$	position vector
$\vec{v}$	velocity vector
C	concentration
$c(\vec{\xi})$	geometric coefficient
c_f	heat capacity for fluid phase, $c_f = (\rho c_p)_f$
c_p	specific heat at constant pressure
c_s	heat capacity for solid phase, $c_s = (\rho c_p)_s$
D	mass diffusivity
Da	Darcy number
Eu	Euler number
K	permeability
L	characteristic length
Le	Lewis number
N	buoyancy coefficient
Nu	Nusselt number
p	pressure
Pr	Prandtl number
Ra_p	porous thermal Rayleigh number, $Ra_p = Ra_T \cdot Da$
Ra_S	solutal Rayleigh number
Ra_T	thermal Rayleigh number
Re_p	pore Reynolds number
Sh	Sherwood number
T	temperature
t	time
v_0	characteristic velocity

validated with several numerical simulations for various governing parameters. Alavyoon [8] published a study where the unsteady and steady convection in a vertical enclosure with applied constant fluxes of heat and mass on the vertical walls is considered. Several comparisons between the fully numerical and analytical solutions for different range of governing parameters are reported. Similar study, where the results of steady state calculations in porous media subjected to uniform fluxes of heat and mass, was published by Mamou et al. [9]. Authors investigated the influence of Rayleigh number, Lewis number and buoyancy coefficient on overall heat and mass transfer in porous enclosure. A comprehensive numerical study was published by Goyeau et al. [10] focusing on the situations with cooperating thermal and solutal buoyancy forces. Nithiarasu et al. [11] studied double-diffusive natural convective flow within a rectangular enclosure with prescribed values of temperature and concentration on the vertical walls, using the generalized porous medium approach. Karimi-Fard et al. [12] investigated the influence of different flow models for porous media on an example of double-diffusive natural convection in a square porous cavity. Bennacer et al. [13] studied the same configuration filled with anisotropic porous media. Recently, Kramer et al. [14] published a numerical study with several results of double-diffusive convective flow in porous cavity obtained with the boundary element method. However, the boundary element method has already been used for the simulations of steady state natural convection problem in porous media, with dual reciprocity transformation of the domain integrals, namely for the Darcy model in Šarler et al. [15] and furthermore, for the Darcy-Brinkman model in Šarler et al. [16].

All of the above mentioned studies are confined on the examples of two-dimensional problems. The studies considering three-dimensional analysis of convective flow due to combined action of thermal and solutal buoyancy forces in different porous media configurations are limited. Sezai and Mohamad [17] published a study, where a three-dimensional geometry of porous media domain is considered, focusing on the case with the opposing effects of buoyancy forces. The results show three-dimensional behavior of the flow for certain parameter ranges with secondary flow formations which can not be captured with two-dimensional models. Similar study presenting some results of the opposed action of thermal and solutal buoyancy forces was published recently by Kramer et al. [18]. Furthermore, the effects of lateral aspect ratio on three-dimensional geometries were studied by Mohamad et al. [19].

The present paper considers a problem of double-diffusive natural convection in a three-dimensional porous media domain using the boundary element method solver. The mathematical model is given at the beginning which is based on the Navier-Stokes equations suitable modified for the simulations of porous media flow, where the Brinkman-extended Darcy formulation is used as a momentum equation. In addition, the boundary element method is briefly outlined. Since all of the above mentioned existing studies are limited in a manner that they are presenting the results of the classical Darcy model and are focused on the effects of opposing gradients of temperature and concentration, in this paper the numerical results of three-dimensional model, revealing the influence of different values of Darcy number and buoyancy coefficient (for the cases of opposing as well as cooperating temperature and concentration gradients) at fixed values of thermal Rayleigh number and Lewis number on the convective motion regime, are studied. The influence of the parameters on the occurrence of three-dimensional flow patterns is investigated.

2. Mathematical model

The problem under consideration is double-diffusive natural convection in a three-dimensional cavity, filled with porous medium which is fully saturated with binary fluid (e.g. aqueous solutions). Two opposite vertical walls are maintained at different temperatures (T_1 and T_2) and concentrations (C_1 and C_2), while the rest of the walls are adiabatic and impermeable.

The mathematical description of the problem is based on the conservation laws for mass, momentum, energy and species concentration, primarily written at the microscopic level, describing the pure fluid flow. In general, that kind of model is not appropriate to describe the fluid flow within the porous media domain, since the geometry is irregular and complex. With an averaging procedure over the representative elementary volume (REV) all flow quantities can be written as volume-averaged values, which enables amenable theoretical treatment. Every macroscopic variable is defined as an appropriate mean