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A meshless approach to non-local damage modelling of concrete

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ABSTRACT

A non-linear continuum damage mechanics model for concrete constructions is analysed using a radial point interpolation meshless method (RPIM). The fundamental mathematical relations and the material model are fully characterized. The 2D plane stress RPIM formulation is extended to a rate-independent standard (local) damage model considering both tension and compression static states. Additionally, in this work, the local damage formulation is modified considering a non-local constitutive damage criterion with regard to a Helmholtz free energy potential. Here, the internal variational fields, such as local and non-local damage variables, are determined by a return-mapping damage algorithm. Due to the non-linear nature of the phenomenon, a displacement controlled Newton-Raphson iterative approach is adopted to attain the non-linear damage solution. In the end, the performance of the proposed non-local damage model is evaluated using an experimental test of a notched-three-point bending beam available in the literature. The obtained solution shows that the meshless methods are capable to effectively analyse concrete structures assuming a non-linear non-local continuum damage model.

1. Introduction

Nowadays, the industrial structural design relies mainly on the finite element method (FEM) [1] analysis to obtain efficiently accurate solutions. This status quo can be explained with the vast number of commercial FEM software packages available in the market and with the robustness and reliability of the FEM. However, this fact cannot hide some of the FEM drawbacks, such as the solution dependency on the element mesh discretization (due to the mesh-based interpolation) or the reduced continuity of its shape functions.

Numerous challenging fields in computational mechanics involve the study of the numerical non-linear local damage solution of concrete materials using FEM formulations, see e.g. [2–9]. Distinct aspects of the continuum damage mechanics field have been investigated by researchers to attain the experimental solution [9,10].

The standard –local– damage mechanics theory is applicable to analyse several problems as introduced by Kachanov [11]. Different types of material experiencing brittle or ductile behaviours could be studied through continuum damage mechanics theory [see i.e. Krajcinovic et al. [12,13]; Resende et al. [14]; Voyiadjis et al. [2]]. The foregoing authors have precisely focused on the local damage mechanics in concrete structures. Afterwards, mathematical relations for rate-independent damage mechanics associated with the local damage were presented by Crisfield [15], Cervera et al. [3–5,16]. The continuum rate-independent damage formulation considered in this study respects Crisfield's and Cervera's hypothesis.

Basically, the degradation of the constitutive model due to the presence of tensile and compressive enforced displacement states include various principal stress terms in standard damage models [8]. As a fact, the standard local constitutive models are inappropriate whenever strong strain softening is encountered. Thus, the governing differential relationships might lose ellipticity.

Numerically, this situation appears itself by spurious mesh sensitivity of finite element computations. As the mesh is refined, the strain starts to localize into a narrow band whose width depends on the element size and tends to zero. Hence, the corresponding relation of load-displacement always experiences snapback for a sufficiently fine mesh, and the total energy dissipated by fracture converges to zero [17].

In FEM studies, the most trustful approach to tackle the aforementioned disturbance, is to regulate the post-peak slope of the stress-strain curve as a function of the element size. Consequently, the energy dissipated in a band of cracking elements will be independent from the bandwidth. It is possible to find in the literature more refined strategies guaranteeing the corresponding objectivity (*so*-called localization limiters) including higher-order gradient models, see e.g. [18–22] and also

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for viscoplastic regularization [23]. The concept of non-local averaging is sufficient for the localization limiters and it can be applied to any kind of constitutive model. The idea of non-local continuum models was proposed by Eringen [24] and later on it was developed for the strain-softening materials by Bazant [25]. Afterwards, Pijaudier-Cabot proposed an improvement leading to the non-local damage theory [26]. Its early extension was established into various approaches of non-local models for damage and fracture mechanics by Jirásek [17]. Furthermore, non-local plasticity combined with the finite element method was presented by Stromberg et al. [27].

Recently, there are other works available in the literature regarding the application of non-local damage formulations, such as: on nano-composites and composite laminated panels [28,29]; concrete materials [30]; complex microstructures [31], dynamic ductile fracture problems combined with an Extended FEM (XFEM) viscoplasticity model [32] and also in manufacturing simulations [33].

It is possible to find some experimental tests reporting the behaviour of concrete materials, such as the softening response of concrete under monotonic uniaxial tension test [34], the behaviour of concrete under compressive enforced displacement [35], the response of concrete under biaxial stress states [36] and the three-point-bending tests on single-edge notched beams [37].

Various demanding isotropic non-linear damage models for concrete structures were analysed with the FEM formulations, such as linear elastic models, see e.g. [9,38,39], rate-dependent models [3,5], viscous-damage models [16] and, more recently, elasto-plastic damage models for crack propagation using XFEM formulations [40].

However, the state-of-the-art lacks a reliable work on meshless methods combined with a non-linear damage constitutive model, particularly whenever the non-local model is required.

Therefore, this work aims to fulfil a gap in the RPIM state-of-the-art, presenting an extensive and complete numerical study of the RPIM regarding the analysis of continuum damage mechanics theory. In the literature, it is possible to find relevant research works combining meshless methods with damage mechanics [41,42]. Nevertheless, those works use particle methods, such as the smoothed particle hydrodynamics (SPH) and reproducing kernel particle methods (RKPM). These formulations, in opposition to RPIM, lack the delta Kronecker property. The efficient aspects of the non-local and standard damage models are fully addressed. Since the authors fully developed their original code to analyse the non-linear phenomenon here proposed, all the relevant attributes of the RPIM can be studied with detail and validated for the proposed damage constitutive model, such as: the size of the influence domain; the integration scheme; the internal fields and optimized damage variables; and the global efficiency.

The present formulation is suited for static or quasi-static structural applications, since inertia is not accounted in this continuum local damage model.

This work is organized as follows; first, in Section 2, the Radial Point Interpolation Method (RPIM) is presented, including the integration scheme, the nodal connectivity, radial point interpolators. Additionally, the 2D elastic solid mechanics formulation for the plane stress state combined with the RPIM, assuming the Galerkin weak form, is shown. In Section 3, first, the rate-independent standard damage theory is formulated based on a Helmholtz free energy function. Furthermore, the foregoing damage formulization is extended to the non-local fashion. Besides, the non-linear return-mapping algorithm of the numerical implementation is also introduced in Section 3. In Section 4, notched-three-point-bending concrete beams are considered and solved with the proposed computational approach. Convergence studies are performed aiming to accomplish the optimum non-local damage characteristics. Then, the results obtained are compared to the experimental solution. The manuscript ends with the final remarks and conclusion, in Section 5.

2. Meshless method

In this work, an advanced discretization meshless technique, the Radial Point Interpolation Method (RPIM), is exerted [43,44]. The RPIM is an interpolator meshless method and forces the nodal connectivity using the influence domain concept. The background integration mesh (required to integrate numerically the integro-differential equations) is constructed using a background regular lattice filled with integration points respecting the Gauss Legendre quadrature principle.

In meshless methods the nodal cloud discretizing the problem domain does not form a mesh, because no previous information regarding the relation between each node is required to construct the interpolation functions [41].

2.1. Numerical integration and nodal connectivity

Similar to FEM, meshless methods are classified in discrete numerical approaches. In meshless methods the domain is discretized with a nodal distribution $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\} \in \Omega \, \Lambda \, \mathbf{x}_i \in \mathbb{R}^2$, being N the total number of nodes discretising the problem domain. Then, a background integration mesh, is constructed, $\mathbf{Q} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_Q\} \in \Omega \, \Lambda \, \mathbf{x}_I \in \mathbb{R}^2$, being Q the total number of integration points discretising the problem domain. Within the RPIM, the integration mesh is completely independent on the nodal mesh and it is required to numerically integrate the integro-differential relations governing the studied phenomenon, the weak-form of Galerkin. As it is well-known, in the FEM formulation, the background integration mesh is constructed using the geometry of the elements and it can be accurately defined because the order of the polynomial shape functions, obtained from the nodal arrangement of the assumed finite element, is known. Hence, it is feasible to determine the number of integration points existing in any integration cell in advance accurately [1,44,45]. However, since in meshless methods the degree of shape function is undisclosed, this predefinition is out of question.

Nevertheless, Wang et al. [46] suggested an integration scheme acceptable for RPIM formulations. First, the problem domain is divided in a regular lattice, forming a quadrilateral grid, with no voids or overlaps. Then, each quadrilateral is filled with integration points respecting the Gauss-Legendre quadrature principle [43]. The literature shows that each integration cell should contain 3 × 3 integration points for a nodal arrangement as the one presented in Fig. 1.

Regarding the nodal connectivity, the RPIM uses the influence-domain concept [43]. This concept implies that each integration point $\mathbf{x}_I$ has to search in the problem domain for the n nearest nodes. Therefore, each integration point $\mathbf{x}_I$ will possess its own influence-domain. This concept will permit to construct the shape functions of each integration point $\mathbf{x}_I$. This issue is described with detail in the literature, see e.g. [43,44].

The performance and accuracy of the final solution is strongly influenced by the size of the influence domains. Thus, it is important to allocate approximately the same number of nodes on each influence domain [43]. In the literature, it is possible to find some meshless

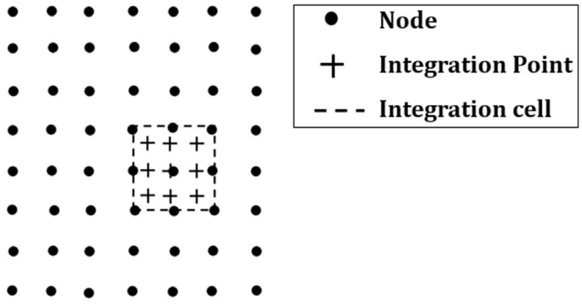

Fig. 1. - An integration cell with 3×3 integration points in the discrete model [45].