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An interaction integral and a modified crack closure integral for evaluating piezoelectric crack-tip fracture parameters in BEM

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ABSTRACT

To evaluate the crack-tip field intensity factors of a piezoelectric crack with any inclined angle, the current widely-used interaction integral method (I-integral) is here extended to the boundary element applications under some coordinate transformations. As well, a new modified crack closure integral method (MCCI) is proposed by considering the discontinuous quarter-point singular elements for the crack-face discretization arising from the dual boundary element method (BEM). This dual BEM involves the strongly singular displacement boundary integral equations (BIEs) for the external boundary and the hypersingular traction BIEs for the crack faces. The crack-tip fracture parameters evaluated by the I-integral and MCCI are verified by the existing analytical solutions and meanwhile, compared with those results achieved by the classical displacement extrapolation method and the J-integral. Three examples are presented to show the high accuracy of the interaction integral method and the improvement of MCCI for the piezoelectric crack problems.

1. Introduction

From the viewpoint of fracture mechanics, of importance is the near-tip field which can be characterized as field intensity factors. For a piezoelectric crack, they include the classical mechanical stress intensity factors (SIFs) and additional electric displacement intensity factor (EDIF), which play a vital role in the design of the smart materials in engineering applications [16].

As we know, the field intensity factors can be obtained analytically for some fundamental crack problems with regular geometrical configuration under simple loading conditions ([9,19,31]). Further the solutions to general crack problems may resort to some efficient numerical methods involving the finite element method (FEM), the extended FEM (XFEM), the boundary element method (BEM), various meshless methods, etc. As a semi-analytical method and dimensionality reduction, the BEM has some significant advantages over other numerical techniques in dealing with crack problems [6]. Meanwhile, the extended FEM pioneered by [1] in terms of the partition of unity has achieved considerable success in dealing with boundary value problems with discontinuities in the last decades. No matter which numerical method is used, the most important and key task is to evaluate the relevant field intensity factors efficiently and accurately. Thus the singularity of the mechanical and electrical fields at the crack-tips must be described somehow in the formulations.

Based on Stroh's formalism to treat dislocations and line charges, the displacement jumps and the voltage across the crack faces can be modeled by a continuous distribution of dislocations [19]. Then the jumps can be related to the field intensity factors, which is well-known as the displacement extrapolation method (DEM). In the same way, the asymptotic singular stress and electric induction fields on the crack plane can be expressed as functions of the field intensity factors referred to as the stress matching method [14,2]. The crack-tip stess field behaves a common square root singularity which results in the difficulty of effectively numerical computation. The crack-tip tractions and charge are always approximated by numerical interpolation which influence the accuracy of this method. So the stess matching method was seldom used in numerical applications. In contrast, the crack opening displacements can be directly and efficiently obtained by any direct BEM. Because of its simplicity and effectiveness, DEM has been the basis to determine the field intensity factors in BEM [12,20,22,5,8]. These two techniques are directly based on the field quantities on the boundary or internal points.

Other than the direct techniques, there are still some energy based methods existing. Based on the concept of the energy-momentum tensor and conservation laws for an elastic plane, the well-known path-independent J-integral was obtained by Rice [25] and further extended to anisotropic piezoelectric problems by Pak [18] and [31], which is verified to equal to the crack-tip energy release rate (ERR). The contour

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integral appearing in the J-integral cannot be accurately evaluated for any domain-type numerical methods such as FEM, because it is highly sensitive to the element meshing. A factitious continuous function should be introduced to transform this contour integral into a domain integral. This crack-tip ERR can be related to the field intensity factors by an Irwin relation. But is not adequate to decouple them for any mixed-mode fracture problems or multi-field coupling systems because of the arising of some additional electric or thermal modes. Then Nishioka et al. [17] proposed a component separation method referred to as J-integral method. But analogous to DEM, the accuracy of the extraction is basically affected by the effectiveness of the numerical results of the crack opening displacements [12]. To avoid this, the interaction integral method was first proposed by Stern et al. [29] to solve mode-I and mode-II SIFs separately for 2D static mechanical problems. Wang et al. [33] extended this method to anisotropic solids. By a superposition of the actual state with an appropriate auxiliary state of analytical solutions, the interaction integral (I-integral) is arisen for extracting the intensity factors. Compared with J-integral, the I-integral has generated a great interest for its convenience in decoupling the field intensity factors. Recently, this method was extended to homogeneous piezoelectric media [11,7] and further to nonhomogeneous piezoelectric materials [24,34]. This technique is also well suited to the BEM due to the ability of the method to compute accurately the internal fields. Aliabadi and his group [21] used the J-integral in the dual BEM as an accurate technique to compute mixed mode stress intensity factors in isotropic materials and further extended to anisotropic plates [28]. But to the authors' knowledge, this method has been seldom exploited in the boundary element procedure to obtain the field intensity factors of piezoelectric cracks [12].

An alternative method to obtain the total ERR is based on a crack closure integral. This technique is first implemented in the FEM by Rybicki and Kanninen [26]. A linear variation of the displacement field around the crack tip was assumed and, consequently, the element ensures a constant strain field. They termed the method as modified crack closure integral technique (MCCI). Later many investigators have employed it and shown its effectiveness in accurate calculation of the SIFs, even for mixed mode crack problems. Local smooth technique was proposed by Ramamurthy et al. [23] and Sethuraman and Maiti [27] imposed on the crack line displacement and stresses using the nodal data. Matti et al. [13] has adapted the MCCI procedure into the BEM for the evaluation of the SIFs in elastic materials. Mukhopadhyay et al. [15] extended this method to thermal crack problems and Lei et al. [12] to piezoelectric crack problems. But from the results, we can found that the MCCI method can achieve an accuracy of around 3–5% for thermal crack problems using either quadratic or quarter point element [15] and about 5–6% for piezoelectric cases [12]. This accuracy is much worse than J-integral method or DEM.

In this paper, the dual BEM based on the strongly singular displacement boundary integral equations (BIEs) for the external boundary together with the hypersingular traction BIEs for the crack faces are used to study some crack problems in piezoelectric materials. The electrical boundary conditions on all crack-faces are assumed to be impermeable. A collocation method is adopted for the spatial discretization. Discontinuous quarter-point elements are integrated into the formulation to capture the crack-tip behavior. The I-integral technique is exploited in this BEM to evaluate the fracture parameters of piezoelectric cracks. Additionally, the MCCI presented in Lei et al. [12] is further modified according to the discontinuous quarter point elements. For any slanted crack, some necessary coordinate transformations are detailed and compared for these methods. The results are verified by the existing analytical solutions and compared with those evaluated by the classical DEM and J-integral methods. The effectiveness and accuracy of the I-integral and the MCCI are demonstrated in some examples for evaluating the stress and electric displacement intensity factors.

2. Boundary integral equations for piezoelectric materials

2.1. Governing equations and boundary conditions

Let us consider a two-dimensional (2D), homogeneous, and linear anisotropy piezoelectric solid in the domain Ω with the boundary $\partial\Omega$. To derive the governing equations and the boundary conditions for piezoelectric materials by the variation principle [19], an electric enthalpy density h is defined as

$$h(\varepsilon_{ij}, E_i) = \frac{1}{2}C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \frac{1}{2}\kappa_{ij}E_iE_j - e_{ikl}\varepsilon_{kl}E_i, \tag{1}$$

where C_{ijkl} are the components of the elasticity tensor measured in a constant electric field, κ_{ij} are the dielectric constants measured at constant strain, and e_{kij} are the piezoelectric constants. In Eq. (1), ε_{ij} and E_i are the strain tensor and the electrical field vector defined by the following gradient equations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \tag{2}$$

where u_i and ϕ denote the elastic displacements and the electric potential, respectively.

By taking h to be the Lagrangian function and employing calculus of variation, the constitutive relations

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{lij}E_l,\ D_i = e_{ikl}\varepsilon_{kl} + \kappa_{il}E_l, \tag{3}$$

the equilibrium equations without any body forces or free electrical charges

$$\sigma_{ij,i} = 0,\ D_{i,i} = 0, \tag{4}$$

and the boundary conditions

$$\sigma_{ij}n_i = t_j,\ D_in_i = -q_s \tag{5}$$

are obtained [19], Where t_j and q_s are the applied surface traction and electrical charge, respectively. In Eqs. (1)–(5) and throughout the paper, a comma denotes the partial differentiation and the summation over repeated indices is applied with all Latin indices ranging from 1 to 2.

2.2. Crack model and dual BEM for a piezoelectric crack

Consider an impermeable crack embedded in a poled piezoelectric plate as shown in Fig. 1. The piezoelectric plate is transversely isotropic elastic with a hexagonal symmetry of class 6 mm with the x_2-axis as the poling direction and the $x_1 - x_3$-plane as the isotropic plane.

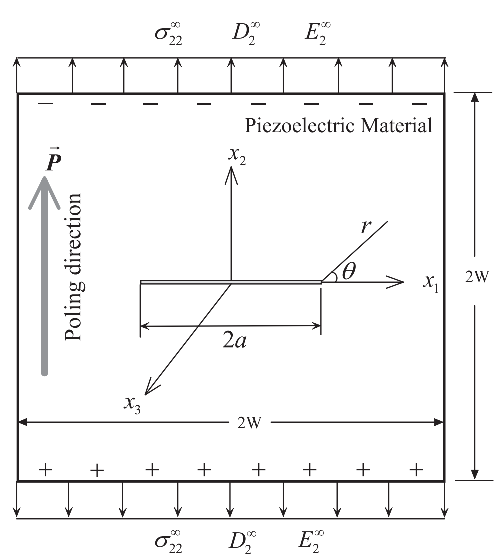

Fig. 1. A Griffith crack under remote loadings σ_{22}^{∞} and D_2^{∞} or E_2^{∞}.