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Thermal shock analysis of 2D cracked solids using the numerical manifold method and precise time integration

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ABSTRACT

The numerical manifold method (NMM), combined with the precise time integration method (PTIM), is proposed for thermal shock fracture analysis. The temperature and displacement discontinuity across crack faces is naturally portrayed attributing to the cover systems in the NMM. The crack tip singularities are characterized through the use of asymptotic bases in the approximations. The discrete equations for transient thermal analysis are firstly solved with the PTIM and then the thermoelastic study is performed. With the interaction integral, the stress intensity factors are computed. Several examples are tested and the nice consistency between the present and existing results is found.

1. Introduction

Engineering equipment such as the aero-engines, gas turbines and pressure vessels is frequently exposed to thermal shock. Under certain conditions, thermally induced stresses may cause the fracture and failure of cracked facilities. Consequently, the study of the behavior of cracked solids under thermal shock is of great importance. In light of the significance, a lot of work has been carried out during the past several decades. Many researchers focused on the analytical solutions. Oliveira and Wu [1] determined the thermal stress intensity factors (TSIFs) of axial cracks in hollow cylinders under thermal shock using the closed form weight function formula. Noda and Ashida [2] adopted the successive approximation, Fourier integral and Bessel series to solve transient thermal annular crack problem in an infinite transversely isotropic cylinder. Lee and Kim [3] calculated the TSIFs of elliptical surface cracks in thin-walled and thick-walled cylinders with the modified Vainshtok's weight function method. Noda and Wang [4] applied the approach of singular integration equation to tackle the thermal shock responses of the functionally graded materials (FGMs) with collinear cracks. Shahani and Nabavi [5] used the finite Hankel transform and the weight function method to compute the TSIFs of internal semi-elliptical crack in a thick-walled cylinder subjected to transient thermal stresses. Wang and Li [6] analyzed the transient thermoelastic fracture of piezoelectric material with the Laplace transformation and integral equation method.

Although widely applied, analytical solutions are generally limited to simple configurations (e.g., single or periodic cracks in infinite or semi-infinite mediums under regular initial or boundary conditions). For problems with finite dimensions, arbitrary cracks or under complex loadings, numerical tools such as the finite element method (FEM), the boundary element method (BEM) and the extended finite element method (XFEM) are much more popular. Emmel and Stamm [7] computed the transient TSIFs for cracked rectangular plate and hollow cylinder with the FEM. Magalhaes and Emery [8] inspected the effects of transient thermal loads on crack propagation in brittle film-substrate structure by the FEM. Through the three-dimensional elastic-plastic FEM, Kim et al. [9] evaluated the integrity of vessel with subclad crack under pressurized thermal shock. Prasad et al. [10] applied the dual BEM and path-independent J-integral to investigate the two-dimensional (2D) transient thermoelastic crack problems. Considering crack closure conditions, Giannopoulos and Anifantis [11] obtained both steady and transient TSIFs of 2D bimaterial interfacial cracks by the BEM. With the uncoupled thermoelastic theory, Zamani and Eslami [12] implemented the XFEM to model the effect of both mechanical and thermal shocks on 2D cracked solids. Rokhi and Shariati [13] studied the response of cracked FGMs under thermal shock with the XFEM in the framework of coupled thermoelasticity.

In the past two decades, considerable efforts have been put on to the development of the numerical manifold method (NMM) proposed by Shi [14]. The soul of the NMM lies in the use of finite cover concept, which has been adopted by Bathe and his coauthors [15–17] to improve the FEM solutions most recently. Benefiting from the use of

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dual cover systems (i.e., the mathematical cover and the physical cover) [14,18], the NMM is very powerful in discontinuous analysis (e.g., in solving crack or inclusion problems). The major highlights of the NMM for crack modeling can be summarized in four aspects: (1) the mathematical cover can be independent of all domain boundaries including cracks; (2) the discontinuity of physical field across crack faces can be manifested in essence; (3) the local property at crack tip zone can be well captured through the use of associated local functions in the approximation, and (4) higher-order approximations can be achieved through the use of higher-order local functions on a fixed mathematical cover. To date, the NMM has been successfully improved to solve various stationary cracks and crack growth problems in homogeneous [19–30] and heterogeneous materials [31–35].

In the present paper, the NMM is further extended to tackle 2D stationary crack problems under thermal shock. The solution procedure is generally divided into two parts: Firstly, the transient heat diffusion problem is analyzed and the corresponding NMM discrete equations are solved with the precise time integration method (PTIM) [36], which has absolute stability, immunity to oscillations and time-step-independent precision. Subsequently, the calculated temperature fields at selected instants are imported into the thermoelastic part to extract the TSIFs.

The remaining of the paper is addressed as follows. In Section 2, the governing equations and associated boundary and/or initial conditions are provided. The NMM formulations for both transient heat conduction and thermoelastic analysis are derived in Section 3. Section 4 presents the details of the PTIM for transient heat conduction analysis and the spatial integral scheme as well. To verify the proposed method, several numerical examples are tested in Section 5. Finally, the concluding remarks are addressed in Section 6.

2. Statement of problems

As shown in Fig. 1, consider a cracked isotropic homogenous physical domain Ω enclosed by the boundary Γ in the unidirectional coupling transient linear thermoelasticity (i.e., the thermal loading affects the displacement, strain and stress fields, but not vice versa). Ignoring both the heat source and the body force, the governing equations for this problem are [10]

$$\rho c\frac{\partial T(\mathbf{x},t)}{\partial t}+\nabla\mathbf{q}(\mathbf{x},t)=0 \tag{1}$$

$$\nabla\cdot\boldsymbol{\sigma}=\mathbf{0} \tag{2}$$

where ρ is the density and c is the specific heat at constant pressure. ∂ denotes partial derivative. $T(\mathbf{x},t)$ is the temperature with $\mathbf{x}\in\Omega$ and t the time. The heat flux $\mathbf{q}$ is determined by the Fourier's law as

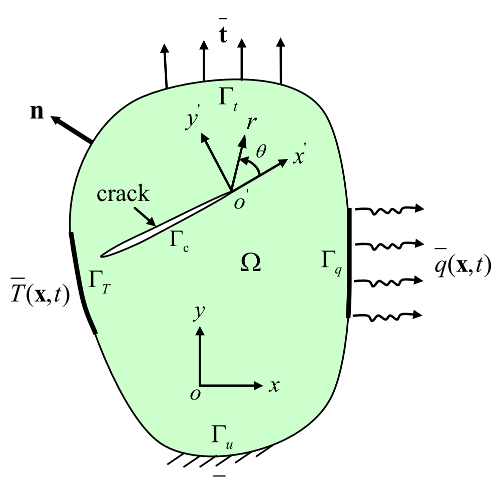

Fig. 1. A cracked homogeneous body in 2D transient linear thermoelasticity.

$\mathbf{q}=-k\nabla T$ with k the thermal conductivity. The stress tensor $\boldsymbol{\sigma}$ is expressed by the generalized Hooke's law as $\boldsymbol{\sigma}=\mathbf{C}:(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}_T)$ with $\mathbf{C}$ the fourth-order Hooke tensor, $\boldsymbol{\varepsilon}=\nabla_s\mathbf{u}$ the total strain tensor and $\boldsymbol{\varepsilon}_T=\alpha(T-T_R)\mathbf{I}$ the thermal strain tensor. $\mathbf{u}$ is the displacement vector, α is the thermal conductivity and T_R is the reference temperature. ∇, ∇_s and $\mathbf{I}$ are, respectively, the gradient operator, the symmetric gradient operator and the second order identity tensor.

The associated boundary conditions for Eqs. (1) and (2) are

$$T(\mathbf{x},t)=\bar{T}(\mathbf{x},t\quad(\mathbf{x}\in\Gamma_T)) \tag{3}$$

$$\mathbf{q}(\mathbf{x},t)\cdot\mathbf{n}=\bar{q}(\mathbf{x},t)\quad(\mathbf{x}\in\Gamma_q) \tag{4}$$

$$\mathbf{u}(\mathbf{x})=\bar{\mathbf{u}}(\mathbf{x})\quad(\mathbf{x}\in\Gamma_u) \tag{5}$$

$$\boldsymbol{\sigma}(\mathbf{x})\cdot\mathbf{n}=\bar{\mathbf{t}}(\mathbf{x})\quad(\mathbf{x}\in\Gamma_t) \tag{6}$$

with Γ_T the temperature boundary, Γ_q the flux boundary, Γ_u the displacement boundary and Γ_t the traction boundary $(\Gamma_T\cup\Gamma_q=\Gamma_u\cup\Gamma_t=\Gamma,\ \Gamma_T\cap\Gamma_q=\Gamma_u\cap\Gamma_t=\varnothing$. The crack boundary Γ_c is a part of natural boundary and is assumed to be adiabatic in heat conduction analysis and traction-free in elastic analysis. $\bar{T}$, $\bar{q}$, $\bar{\mathbf{u}}$ and $\bar{\mathbf{t}}$ are, respectively, the prescribed temperature, flux, displacement and traction on corresponding boundary. $\mathbf{n}$ is the outward unit normal to the domain.

The initial condition for Eq. (1) is

$$T(\mathbf{x},0)\ =T_0\quad(\mathbf{x}\in\Omega) \tag{7}$$

3. Thermal shock fracture analysis by the NMM

3.1. NMM approximations

In the NMM, to solve a crack problem, the mathematical cover (MC), composed of a series of mathematical patches (MPs), is firstly built. Broadly speaking, the MP, formed by mathematical elements, can be of any shape and overlapped. The MC may be independent of all domain boundary (including cracks) but must be large enough to cover the whole cracked domain. On each MP, a partition of unity (PU) [37] weight function is defined. Next, the physical patches (PPs), the collection of which is termed as the physical cover (PC), are formed by the intersection of MPs and physical domain. On each PP, the local function is constructed to represent the local physical property. Then, the manifold elements (MEs) are generated through the shared region of PPs. To make the above concepts and procedures clear, an illustrated example in Fig. 2 is further adopted.

Consider the physical domain $\Omega=\Omega_1\cup\Omega_2$ in Fig. 2a. The MC consisting of two rectangular MPs, i.e., M_1 and M_2 in Fig. 2b, is chosen to cover the whole domain. The intersection of the physical domain and the MPs gives 4 PPs in Fig. 2c, that is, P_1 and P_2 from M_1, and P_3 and P_4 from M_2. Above 4 PPs finally generate 7 MEs, numbered e_1, e_2..., and e_7 in Fig. 2d, where the bracketed contents represent the associated PPs.

Following the aforementioned process, we can obtain the NMM approximation on each ME by pasting the local functions using the associated weight functions. Accordingly, for the present problem, the temperature and displacement field on any ME e is approximately expressed as

$$T^h(\mathbf{x},t)=\sum_{i=1}^{n_t}w_i(\mathbf{x})T_i(\mathbf{x},t) \tag{8}$$

$$\mathbf{u}^h(\mathbf{x})=\sum_{i=1}^{n_t}w_i(\mathbf{x})\mathbf{u}_i(\mathbf{x}) \tag{9}$$

where n_t is the amount of PPs shared by e. $w_i(\mathbf{x})$ is the PU weight function defined on the MP containing the ith PP. $T_i(\mathbf{x},t)$ and $\mathbf{u}_i(\mathbf{x})$ are,