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The birth of the boundary element method from conception to application

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ABSTRACT

The Boundary Element Method (BEM) has now become a well established numerical technique with a number of computer programmes to its credit oriented towards industrial applications. They are reliable and robust tools which stress its unique features versus finite elements, ie reduced dimensionality which makes it easier to interface it to CAD codes; better accuracy, the elegant way in which moving boundaries are dealt with and the possibility of taking into consideration infinite domains without the need of introducing artificial boundaries. An excellent paper on the early work that led to the development of boundary elements has been given by A & D Cheng [1] and from which many of the illustrations depicting famous scientists were taken. The present contribution aims to explain further how the methodology developed and consolidated towards the end of the 1970s, beginning of the 1980s.

The current state of the Method is the result of the work of a large number of researchers and software specialists. It has been a long and very successful undertaking since 1978 which is perceived to be the birth date of the technique, ie when the first conference on BEM was held at Southampton University. It was also then that the first book on the Method was published. The fundamentals and methodology, still those in use, were established at that moment.

To understand how the idea of the Boundary Element Method developed, one needs to refer to the state of the art in finite elements (FEM) in the 1970s. That method was based in terms of the minimisation of a functional such as the corresponding to minimum potential energy or similar. This was the basis of the so-called Rayleigh (1842–1919) and Ritz (1874–1909) technique (Fig. 1). Later on, the idea that functionals were not needed gave rise to the use of the Galerkin (1871–1945) Method, an idea that took some time to be accepted [1].

Although the history of Boundary Elements is largely independent of that, it uses similar shape functions as FEM for the distribution of the variables over the surface. Its basis however can be traced to integral equations theory.

The author's own introduction to integral equations came through the work of the Italian School, the influence of which was strong in latin countries. It included such outstanding scientists as Enrico Betti (1823–1892), renowned for his reciprocity principle and a source of inspiration for others such as Carlo Somigliana (1890–1955), whose identity for stress analysis can be directly applied to obtain BEM formulations and Vito Volterra (1860–1940) (Fig. 2). The last scientist originated a type of integral equation which could be used for time-dependent problems, including modelling of visco-elastic effects in concrete. Volterra's equations were applied in the author's first research laboratory (University of Rosario, Argentina) to study creep buckling of concrete columns and the results compared against a series of experiments. At that time, that work was carried out under Professor Nestor Distefano (1931–1975) late of the University of California at Berkeley, where he was to die suddenly at an early age, before he was able to develop his outstanding capabilities to their full potential. Nestor was exceptionally intelligent as well as possessing an inquisitive mind; always at his best when working in a multi-disciplinary environment.

It was after moving to the UK that the author started to appreciate the advantages of more general integral equation formulations and in particular the work of George Green (1793–1841) on setting up the basis for direct integral methods and Erik Fredholm (1866–1927), who contributed to the theory of indirect integral equations (Fig. 3).

BEM is nowadays mostly based on the direct formulation and hence owes much to the pioneering work of George Green. He published his first and most influential papers on a subscription basis; ie for a group of people prepared to buy each new article, something that could only have taken place in the British Enlightenment period, when society as a whole was closely following all major scientific developments. The fact is that few of his subscribers were able to follow Green's work and his ideas lay dormant until rediscovered by Lord Kelvin, who understood their importance.

It was also due to the influence of several other mathematical and engineering scientists that the BEM was born. The original fundamentals came from the Southampton University Group where the author was carrying out his research, inspired by his Supervisor, Professor Hugh Tottenham (1926–2012), a brilliant researcher who unfortunately left very few publications behind. He was the inspiration of the whole group working on engineering mechanics at the Department of Civil Engineering, and his influence was marked in the field of integral equations (Fig. 4).

Tottenham studied at Cambridge at a time when many intellectuals saw Russia and the communist regime in a favourable light. He learnt Russian during his studies and was able to convey to his disciples the work of many mathematicians working in that country on integral equations at a time when there were few translations available. This referred specially to the Georgian School, represented by Nikolai Muskhelishvili (1891–1974) and Viktor Kupradze (1903–1985), the latter one of the more important references on direct boundary integrals. Tottenham served until his death as a member of the Board of Directors of the Wessex Institute of Technology (WIT), where work on BEM was to continue until now.

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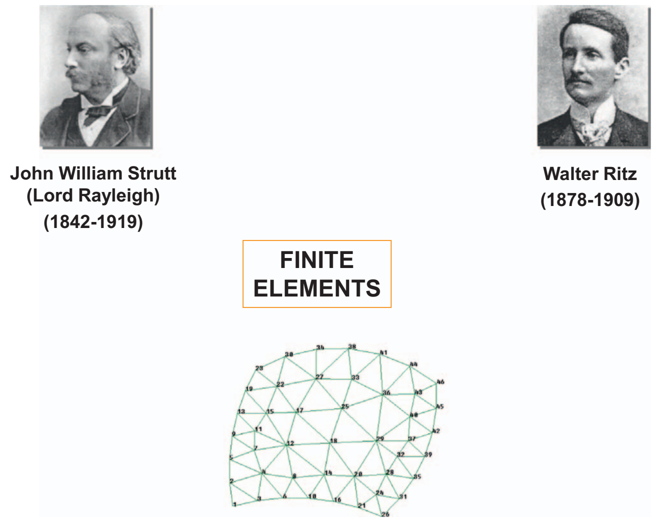

Fig. 1. The FEM was originally expressed in terms of Rayleigh Ritz Method, a technique based on the minimisation of a function.

Fig. 2. The Italian School of mechanical sciences, which led to Somigliana's identity and the Volterra integrals.

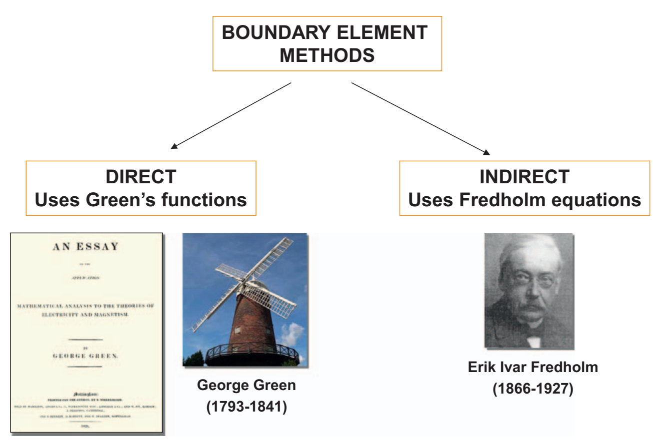

Fig. 3. The integral equation formulations originated the direct and indirect BEM.

Parallel to Tottenham's work, Prof Maurice Jaswon (1922–2011) of City University was also developing boundary integral formulations. His research gave origin to a book with George Symm (until a few years ago also a member of WIT's Board of Directors). The work of this Group somewhat petered out, possibly on the assumption that the contemporary emergence of FEM made any further research on other methods unnecessary. It was a mistake on their part.

An important by-product of Jaswon's work was the setting up of boundary integral equation research in the USA under Frank Rizzo (1938). This was the result of Maurice's time spent there when Rizzo realised the possibility of applying Somigliana's identity to obtain boundary integral formulations for stress analysis. A series of bright researchers were influenced by Rizzo, including Thomas Cruse (1941). Their excellent work has unfortunately not been continued.

At the end of 1960, beginning of 1970, the fundamentals of boundary integral equations and the parallel development of the FEM methodology were in place. The moment was right for the emergence of BEM but a further component was required.

This was provided by the work at MIT on mixed formulations; originally due to Eric Reissner (1913–1996) quasi variational principles which were extended by Jerry Connor (1932) from the Civil Engineering Department to finite elements. The author was fortunate enough to take a course with Reissner and even more to work with Jerry Connor, one of the most brilliant researchers the author has ever known. He is also a superb teacher