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Iterative multi-domain BEM solution for water wave reflection by perforated caisson breakwaters

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ABSTRACT

This study develops a full solution for water wave reflection by a partially perforated caisson breakwater with a rubble mound foundation using multi-domain BEM (boundary element method). Regular and irregular waves are both considered. A quadratic pressure drop condition on caisson perforated wall is adopted, and direct iterative calculations are performed. Due to the use of quadratic pressure drop condition, the effect of wave height on the energy dissipation by the perforated wall is well considered. This study also develops an iterative analytical solution for wave reflection by a partially perforated caisson breakwater on flat bottom using matched eigenfunction expansion method. The reflection coefficients calculated by the multi-domain BEM solution and the analytical solution are in excellent agreement. The present calculated results also agree reasonably well with experimental data from different literatures. Suitable values of discharge coefficient and blockage coefficient in the quadratic pressure drop condition are recommended for perforated caissons. The effects of the wave steepness, the blockage coefficient of perforated wall and the relative wave chamber width on the reflection coefficient are clarified. The present BEM solution is simple and reliable. It may be used for predicting the reflection coefficients of perforated caisson breakwaters in preliminary engineering design.

1. Introduction

A perforated caisson breakwater, which consists of a perforated front wall, an impermeable rear wall and a wave absorbing chamber between them, has the merits of lower reflection coefficient and smaller wave force in comparison with a non-perforated caisson breakwater [1,2]. Thus, perforated caisson breakwaters have been often used in port and coastal engineering. For engineering design and application, efficient mathematical models for estimating the reflection coefficients of perforated caisson breakwaters should be of significance. The objective of this study is to develop a simple and reliable solution for wave reflection by perforated caisson breakwaters using multi-domain BEM (boundary element method).

The process of wave reflection and dissipation by perforated caisson breakwaters is very complicated. This may be numerically simulated by solving the Navier–Stokes equation [3–5]. Alternatively, the energy dissipation and the phase shift of wave motion induced by a perforated thin wall can be represented by linear pressure drop conditions [6,7] or quadratic pressure drop conditions involving the effect of wave height [8–11]. By using these pressure drop conditions, simple solutions for wave interactions with perforated caisson breakwaters can be developed based on linear potential theory. For perforated caisson breakwaters located on rubble fill foundations (flat bottoms), all the boundaries of fluid domains coincide with coordinate lines or free surfaces. Thus, one can develop series solutions of velocity potentials by the separation of variables, and determine the expansion coefficients in the series solutions using pressure drop conditions on perforated walls [12–19]. In practice, perforated caissons are often built on trapezoidal rubble mound foundations. For this case, the method of separation of variables generally cannot work due to the sloping bottom boundary. Based on the Galerkin-eigenfunction method and the finite difference method, Suh et al. [20] developed a numerical solution for wave reflection by a partially perforated caisson breakwater on rubble mound foundation. In their solution, the evanescent models in velocity potentials were neglected, and a linearized version of quadratic pressure drop condition [8] was used. In addition, Suh et al. [20] assumed the seaside lower non-perforated part of caisson (vertical face) to be a very steep slope.

In this study, we will develop a full solution for wave reflection by a partially perforated caisson breakwater on a rubble mound foundation using multi-domain BEM. In the multi-domain BEM solution, the whole fluid region is divided into multiple domains according to the geometrical shape of the structure. The integral equation is independently discretized in each domain. The variables in different domains are connected and simultaneously determined using the boundary

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conditions on their common boundaries. The beauty of BEM is that the complicated fluid boundaries can be accurately described, and the full solution of velocity potential near the breakwater (including propagation and evanescent models) can be obtained. An early study on water wave problem using multi-domain BEM can be found in Ijima et al. [21]. They examined wave motions through porous breakwaters. The multi-domain BEM has been often used for solving wave interactions with ocean structures involving porous medium [22–24] or perforated thin plates [25–28]. Besides multi-domain BEM solution, numerical solutions for wave scattering by perforated thin wall breakwaters have also been developed by dual BEM [29] and scaled boundary finite element method [30]. In these BEM solutions, all the boundary conditions were linear without considering the effect of wave height on the wave energy dissipation. In order to well consider the effect of wave height on the wave energy dissipation, the present multi-domain BEM solution will directly solve a quadratic pressure drop condition on the caisson perforated wall [10,11]. Due to the nonlinear pressure drop condition, iterative calculations are needed. However, the solution method of multi-domain BEM can ensure simple and efficient iterative calculations.

To validate the present multi-domain BEM solution in theory, this study will also develop an iterative analytical solution for wave reflection by a partially perforated caisson breakwater on a flat bottom. In most of previous analytical solutions for perforated caisson breakwaters, linear pressure drop conditions or linearized versions of quadratic pressure drop conditions were adopted. But, Molin and Fourest [13] developed an analytical solution for wave reflection by a multi-chamber fully perforated caisson breakwater (wave absorber) by directly solving a quadratic pressure drop condition. In their solution, the water depths inside and outside the structure were the same, and the inertial effect of perforated walls was neglected. Zhu and Chwang [31] also directly used a quadratic pressure drop condition to develop an analytical solution for wave reflection by a semi-immersed perforated thin wall with an impermeable rear wall. To the authors' knowledge, a full analytical solution for wave reflection by a partially perforated caisson breakwater involving quadratic pressure drop condition has not been developed. Such an analytical solution can serve as a benchmark for numerical solutions.

The governing equation and the boundary conditions for the present problem are formulated in the following section. The iterative multi-domain BEM solution and the analytical solution are developed in Sections 3 and 4, respectively. In Section 5, the solutions are extended to considering irregular waves. The two solutions are compared in Section 6. The calculated results by the solutions are compared with experimental data from different literatures. The effects of several parameters on the reflection coefficient are also discussed. Finally, the main conclusions of this study are drawn.

2. Boundary value problem

The idealized sketch for the present problem is shown in Fig. 1. A partially perforated caisson is located on a rubble mound foundation

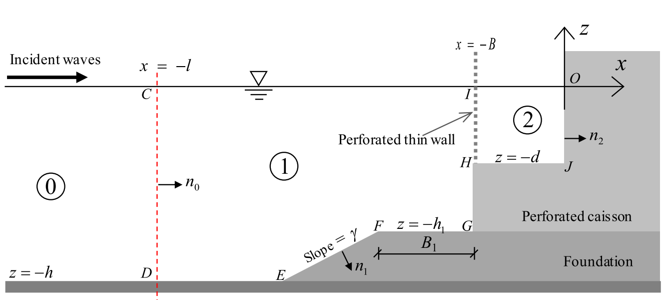

Fig. 1. Idealized sketch for wave reflection by a partially perforated caisson on a rubble mound foundation.

with a seaside slope, and the slope is γ. The whole structure serves as a composite breakwater. The perforated caisson has a wave chamber with width B, and the wave depth inside the wave chamber is d. The berm width of the foundation is B_1, and the water depth on the berm is h_1. The water depth in front of the foundation is h. A Cartesian coordinate, with the origin at the still water level and the z-axis pointing upwards along the chamber rear wall, is adopted for mathematical descriptions. The incident waves from open sea propagate along the positive x-direction.

We examine the present wave-structure interaction problem based on potential theory. That is to say, the fluid is assumed to be inviscid and incompressible, and its motion is irrotational. Then, a velocity potential $\Phi(x, z, t)$ can be used to describe the fluid motion. We further consider linear time-harmonic incident waves, and have

$$\Phi(x, z, t) = \mathrm{Re}\{\phi(x, z)e^{-\mathrm{i}\omega t}\}, \tag{1}$$

where Re denotes the real part of argument, $\mathrm{i} = \sqrt{-1}$, ω is the angular frequency of incident waves, and $\phi(x, z)$ is the complex spatial velocity potential.

In order to develop a multi-domain BEM solution, we introduce a vertical fictional boundary at $x = -l$, as shown in Fig. 1. The outer fluid region on the left-hand side of the fictional boundary is named as domain 0 ($x \le -l$). The inner fluid region on the right-hand side of the fictional boundary is further divided into two domains: domain 1, the fluid region outside the wave chamber (bounded by the curve *CDEFGHIC*); and domain 2, the fluid region inside the wave chamber (bounded by the curve *HIOJH*).

In each domain, the spatial velocity potential satisfies the Laplace equation:

$$\frac{\partial^2\phi_j(x, z)}{\partial x^2} + \frac{\partial^2\phi_j(x, z)}{\partial z^2} = 0, \quad j = 0, \ 1, \ 2, \tag{2}$$

where j denotes the velocity potential in domain j. On the still water level, the velocity potentials satisfy the linearized free surface condition:

$$\frac{\partial\phi_j}{\partial n_j} = \frac{\partial\phi_j}{\partial z} = \frac{\omega^2}{g}\phi_j, \quad z = 0, \ j = 0, \ 1, \ 2, \tag{3}$$

where g is the acceleration due to gravity, n_j denotes the unit normal vector pointing away from domain j. On the left far field, the reflected waves must be outgoing:

$$\frac{\partial\phi_r}{\partial x} = -\mathrm{i}k_0\phi_r, \quad x \to -\infty, \tag{4}$$

where ϕ_r is the velocity potential of reflected waves, and k_0 is the wave number in water depth h. The seabed and the mound foundation are both assumed to be impermeable. Thus, the velocity potentials satisfy the non-penetration boundary conditions:

$$\frac{\partial\phi_0}{\partial n_0} = -\frac{\partial\phi_0}{\partial z} = 0, \quad z = -h, \tag{5}$$

$$\frac{\partial\phi_1}{\partial n_1} = 0, \quad (x, z) \in DEFG. \tag{6}$$

On the caisson impermeable surfaces, the velocity potentials also satisfy non-penetration boundary conditions:

$$\frac{\partial\phi_1}{\partial n_1} = \frac{\partial\phi_1}{\partial x} = 0, \quad (x, z) \in GH, \tag{7}$$

$$\frac{\partial\phi_2}{\partial n_2} = 0, \quad (x, z) \in OJH. \tag{8}$$

On the common boundary CD ($x = -l$), the velocity potential and the fluid horizontal velocity must be continuous:

$$\phi_0 = \phi_1, \quad (x, z) \in CD, \tag{9}$$