Engineering Analysis with Boundary Elements 77 (2017) 81–88

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Modeling of thermo-mechanical fracture behaviors based on cohesive segments formulation

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ARTICLE INFO

Keywords:
Cracks
Thermoelastic
Numerical methods
Cohesive zone
Mesh-free method

ABSTRACT

An element-free framework is developed to study the thermo-mechanical fracture behavior of materials based on the cohesive segments model, in which a crack is treated as a combination of a series of cohesive segments and a new cohesive segment is added whenever the cracking criterion is met at a node. Using the moving least-square shape functions as the partition of unity, the discontinuity field is approximated with extra degrees of freedom at the existing nodes. Cohesive constitutive laws are used to model force and heat transfer through cracks. Mechanical and temperature fields are incorporated into a coupled nonlinear system, and the crack problem is iteratively solved. The chosen numerical examples illustrate the efficiency and flexibility of the proposed method.

1. Introduction

The cracking and failure behavior of materials under thermo-mechanical coupling loads is of high relevance to many applications of engineering and science; for example, it forms the foundation from which to understand the constitutive response of concrete in situations of significant temperature change or fire [1,2]. Linear elastic fracture mechanics has been popularly used in many engineering problems, in which a characteristic singularity dominates the stress and strain fields around the crack tip, and a critical stress intensity factor or energy release rate is used as the criteria for failure. However, the classical fracture mechanics lacks a physics-based description of the failure process, and as such is unable to provide reasonable physical explanations [3,4].

A new alternative approach is the cohesive zone model, which is recognized as an important tool for describing the delamination and cracking of engineering materials [5–7]. In the cohesive zone model, it is assumed that a narrow band of vanishing thickness, known as the cohesive zone, exists ahead of a crack tip, and that this represents the fracture process zone. The cohesiveness of the zone surfaces is created by cohesive traction, which follows a cohesive constitutive law. Crack growth occurs when the separation at the tail of the cohesive zone reaches a critical value, at which cohesive traction disappears.

Based on the cohesive model, Remmers et al. [8] proposed a cohesive segments model to efficiently simulate coexisting crack initiation and nucleation, in which cracking is regarded as the appearance and combination of a series of crack segments. The cohesive segments model has been applied to the numerical simulations of the cracking and failure process by the present authors [9] and other researchers [10,11], which shows a good computational efficiency and flexibility.

The temperature jump between the two faces of a crack has been observed, and the temperature field around the crack tip shows a singularity similar to the displacement field. Similarly, the heat cohesive law can also be introduced to model the crack tip field. Fagerstrom and Larsson presented a thermo-mechanical cohesive zone formulation for ductile fracture [12]. Ozdemir et al. developed a zone formulation that is suitable for the thermo-mechanical analysis of heterogeneous solids and structural systems with contacting/interacting components [13]. Evangelista et al. formulated a three-dimensional thermodynamic cohesive zone model for the fracture of cementitious materials [14]. Hattiangadi and Siegmund developed a thermo-mechanical cohesive zone model for bridged delamination cracks of composite laminates [15]. A central objective of the present study is to combine the cohesive segments model and the thermo-mechanical cohesive constitutive law to establish an analysis method for the cracking and failure of materials subjected to thermo-mechanical coupling loads.

Another key issue associated with the present problem is the treatment of the displacement discontinuities that correspond to the

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http://dx.doi.org/10.1016/j.enganabound.2017.01.007
Received 10 June 2016; Received in revised form 13 October 2016; Accepted 23 January 2017

evolving crack. Recently, the enriched degrees of freedom method has been popularly adopted to overcome this difficulty, which leads to the extended finite element method (FEM) through the embedding of displacement jumps within elements [16–19], and arbitrary crack growth can be modeled without remeshing. This technique has also been combined with the element-free/meshless methods [20–23] to establish extended/enriched element-free methods for the fracture mechanics problems [24–26]. Nguyen et al. present an extended mesh-free Galerkin method for the 2-D quasi-static crack growth with radial point interpolation method [24]. In the enriched mesh-free method, the enrichment can be added to arbitrary nodes and a greater degree of flexibility can be achieved [9,10,25]. In addition, inspired by the strain smoothing technique emerged from the mesh-free method [27], Liu and Nguyen-Thoi have developed a family of smoothed finite element methods (S-FEMs) [28] to avoid the element distortion in the classical FEMs and improve the computational instability and accuracy, which includes the cell-based S-FEM (CS-FEM) [29], node-based S-FEM (NS-FEM) [30], and edge-based smoothed FEM (ES-FEM) [31] and a face-based S-FEM (FS-FEM) [32]. Through constructing the singular crack-tip triangular element, ES-FEM has been applied to fracture mechanics problems [33–35]. Although many methods have their advantages in the computational stability and accuracy, the present work attempts to establish a simple and flexible method by locating a cohesive segment at the fixed mesh-free node at which the failure criterion reaches, and it can also be viewed as an extended research of authors' previous work [9].

2. Governing equations

Consider the domain Ω bounded by the boundary $\Gamma = \Gamma_u \cup \Gamma_t \cup \Gamma_d = \Gamma_\theta \cap \Gamma_q$ with the conditions $\Gamma_u \cap \Gamma_t = 0$, $\Gamma_d \cap \Gamma_t = 0$ and $\Gamma_u \cap \Gamma_d = 0$. Γ_θ refers to the boundary where the temperature is imposed; Γ_q refers to the boundary where the flux is imposed; Γ_q and Γ_q denote the Neumann boundary and Dirichlet boundary, respectively; Γ_d refers to the crack surface. The equilibrium equation and the heat equation in strong form are expressed as [12–15].

$$\sigma_{ij,j} + b_i = 0 \quad \mathbf{x} \in \Omega \tag{1a}$$

$$q_{i,i} + Q = 0 \quad \mathbf{x} \in \Omega \tag{1b}$$

where σ_{ij} denotes the stress tensor, b_i is the body forces, Q is the heat source and q_i denotes the heat flux. For the thermo-mechanical problem, the constitutive equations are written as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2a}$$

$$\varepsilon_{kl} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \alpha_T \Delta\theta \mathbf{I}_{ij} \tag{2b}$$

$$q_i + k\theta_{,i}=0 \tag{2c}$$

where $\Delta\theta$ is the change in temperature, and $\mathbf{I}_{ij}$ is the second order identity matrix. C_{ijkl} is the component of the stiffness tensor. α_T is the expansion coefficient and k is the diffusivity. It is noted that the thermal field influences the mechanical field, while there is no influence of the mechanical field on the thermal field except that the heat conductance coefficient has a dependence on the crack opening. The boundary conditions are given by

$$u_i = \bar{u}_i \quad \text{on } \Gamma_u \tag{3a}$$

$$\theta = \bar{\theta} \quad \text{on } \Gamma_\theta \tag{3b}$$

$$t_i = \sigma_{ij}n_j = \bar{t}_i \quad \text{on } \Gamma_t \tag{3c}$$

$$\bar{q} = q_i n_i \quad \text{on } \Gamma_q \tag{3d}$$

In the case of no body force and no heat source, the weak form of the governing equations are given as

$$\int_\Omega \sigma_{ij}\delta\varepsilon_{ij}\, dV + \sum_{j=1}^{m} \int_{\Gamma_{d,j}} t_i \delta [u]_i d\Gamma = \int_{\Gamma_t} \bar{t}_i \delta u_i d\Gamma \tag{4a}$$

$$\int_\Omega \theta_{,i} k\delta\theta_{,i}\, dV + \sum_{j=1}^{m} \int_{\Gamma_{d,j}} q_C \delta [\theta] d\Gamma = \int_{\Gamma_q} \bar{q}\delta\theta d\Gamma \tag{4b}$$

In which $[\theta]$ is the temperature jump along a crack. In the cohesive segments method, a failure criterion is used to judge when and where a cohesive segment should be introduced [8–10]. A softening constitutive model is then applied at the internal boundary Γ_d to relate the traction vector $\mathbf{t}$ to the jump in the displacement field $[\mathbf{u}]$. The material is assumed to behave in a local nonlinear fashion. In the present paper, an improved exponential cohesive zone law [36,37] is used to describe the traction–separation relationship

$$t_n = \frac{\phi_n}{\delta_n}(\frac{[u]_n}{\delta_n})\exp(-\frac{[u]_n}{\delta_n})\exp(-\frac{[u]_t^2}{\delta_t^2}) \tag{5a}$$

$$t_t = 2\frac{\phi_t}{\delta_t}(\frac{[u]_t}{\delta t})(1 + \frac{[u]_n}{\delta_n})\exp(-\frac{[u]_t^2}{\delta_t^2})\exp(-\frac{[u]_n}{\delta_n}) \tag{5b}$$

where $[u]_n$ and $[u]_t$ define the normal and tangential separations on the crack, respectively; t_n and t_t are the normal and tangential tractions; δ_n and δ_t are the characteristic opening lengths for normal and tangential directions; and ϕ_n and ϕ_t represent the normal and tangential separation work.

A temperature jump between the two faces of the discontinuity of a crack can be observed following the coupling of the interfacial load and heat transfer mechanisms [12,13]. The temperature jump is intimately linked to the interface heat conduction within the crack, and the heat flow through the crack can be expressed as

$$q_C = k_C[\theta] \tag{6}$$

where k_C is the heat conductance coefficient through the crack surface, quantifying the heat transported between the two surfaces of the crack. In the present study, k_C is considered to have a linkage to crack opening displacement as

$$k_C = k_{C0} - E_\theta [u] \tag{7}$$

3. Displacement and temperature fields

In the cohesive segments model, a continuous crack is modeled as a combination of cohesive segments, as shown in Fig. 1, and a new cohesive segment is introduced whenever a loss of continuity criterion is met at a stress integration point or node.

For a body Ω that contains m cohesive segments (Fig. 2), the displacement field can be regarded as a continuous regular displacement field $\mathbf{u}^{\text{con}}(\mathbf{x})$ plus m discontinuous displacement fields $H_{\Gamma_{d,j}}\mathbf{u}_j^{\text{dis}}(\mathbf{x})$ $(j = 1,\ 2,\ \ldots,\ m)$ as follows

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}^{\text{con}}(\mathbf{x}) + \sum_{j=1}^{m} H_{\Gamma_{d,j}}\mathbf{u}_j^{\text{dis}}(\mathbf{x}) \tag{8}$$

where the vector x denotes the position of a material point in domain Ω, $\mathbf{u}^{\text{con}}(\mathbf{x})$ and $\mathbf{u}_j^{\text{dis}}(\mathbf{x})$ are continuous functions of Ω, and $H_{\Gamma_{d,j}}$ is a Heaviside step function that is centered on discontinuity $\Gamma_{d,j}(H_{\Gamma_{d,j}} = 1\text{if}\mathbf{x} \in \Omega_j^+,\ \ H_{\Gamma_{d,j}} = 0\text{if}\mathbf{x} \in \Omega_j^-)$.

The magnitude of the displacement jump at a discontinuity is given by

$$[\mathbf{u}_i] = \sum_{j=1}^{m} H_{\Gamma_{d,j}}[\mathbf{u}_j(\mathbf{x})] \qquad \mathbf{x} \in \Gamma_{d,i} \tag{9}$$

The strain field in the bulk can be obtained by taking the derivative of the displacement field (8). To ensure geometric linearity, the strain tensor is given by