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An implicit potential method along with a meshless technique for incompressible fluid flows for regular and irregular geometries in 2D and 3D

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ABSTRACT

We present the Implicit Potential (IPOT) numerical scheme developed in the framework of meshless point collocation. The proposed scheme is used for the numerical solution of the steady state, incompressible Navier-Stokes (N-S) equations in their primitive variable (*u*-*v*-*w*-*p*) formulation. The governing equations are solved in their strong form using either a collocated or a semi-staggered type meshless nodal configuration. The unknown field functions and derivatives are calculated using the Modified Moving Least Squares (MMLS) interpolation method. Both velocity-correction and pressure-correction methods applied ensure the incompressibility constraint and mass conservation. The proposed meshless point collocation (MPC) scheme has the following characteristics: (*i*) it can be applied, in a straightforward manner to: steady, unsteady, internal and external fluid flows in 2D and 3D, (*ii*) it equally applies to regular an irregular geometries, (*iii*) a distribution of points is sufficient, no numerical integration in space nor any mesh structure are required, (*iv*) there is no need for pressure boundary conditions since no pressure constitutive equation is solved, (*v*) it is quite simple and accurate, (*vi*) results can be obtained using collocated or semi-staggered nodal distributions, (*vii*) there is no need to compute the velocity potential nor the unit normal vectors and (*viii*) there is no need for a curvilinear system of coordinates. Simulations of fluid flow in 2D and 3D for regular and irregular geometries indicate the validity of the proposed methodology.

1. Introduction

One of the problems arising in incompressible flow is the explicit treatment of pressure in equations of motion. Moreover, solving numerically the Navier-Stokes (N-S) equations is a challenging task for a number of reasons. First, and most important, is the inherent nonlinear nature of the partial differential equations. For high velocity or low viscosity the governing equations can produce highly unstable flows (formation of eddies). Second, is the imposition of the incompressibility constraint, with the central question to be answered being the calculation of pressure boundary conditions [1], considering that the governing equations do not provide any boundary conditions for the pressure. Any algorithm developed must ensure a divergence-free flow field at any given time during the calculation.

A significant number of techniques have been developed aiming to deal with the incompressibility constraint [2]. All were successfully incorporated into the traditional mesh-based methods, such as Finite Difference Method (FDM), Finite Element Method (FEM) and Finite Volume Method (FVM). One of the first methods developed using the FDM was the MAC (Marker-and-Cell) scheme, introduced by Harlow and Welch [3]. The MAC scheme is a direct discretization of the (N-S) equations in their primitive variables formulation using second order finite differences on a staggered grid. The convection and viscous terms are solved using explicit time integration, while the pressure term using implicit time integration. Additionally, there is a decoupling of computing the velocity and pressure fields, with the incompressibility constraint being solved on the discretized momentum equation, which results in a discrete Poisson equation for the pressure. In the late 60s

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Chorin [4] introduced the projection method that allows simplified treatment of the viscous term. In the context of projection methods an intermediate velocity is computed first and then projected onto the space of incompressible vector fields by solving a Poisson-type equation for pressure. The first successful application of FEM to flow problems might be the work of de Vries and Norrie [5], where the Galerkin FEM was applied to incompressible flow with low and moderate Reynolds number. Despite its success, in the cases of high Reynolds numbers the nonlinear convective terms induce numerical oscillations. Consequently, the standard Galerkin FEM, known to be unstable in convection dominated regimes, was modified and new sophisticated methods emerged, such as the streamline upwind Petrov-Galerkin (SUPG) method, the sub-grid scale method, the finite increment calculus (FIC) method, the Taylor-Galerkin (TG) method and the characteristic-based split (CBS) method [6,7]. In the context of Finite Volume methods frequently used methodologies belong to the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) family [8,9]. Alternative approaches are the artificial compressibility technique [10] and the Continuity Pressure Vorticity (CVP) method [11,12]. Therein, the velocity field is corrected according to a well-known vector identity and, on the basis of this correction, the pressure field is subsequently updated. The solution is obtained using the Helmholtz decomposition of the velocity vector and a modified Bernoulli's law for the coupling of the velocity-pressure for the simulation of external flows. In [13] a novel auxiliary potential velocity scheme for incompressible flows was presented, while in [14] the implicit potential method was applied utilizing an implicit potential velocity method for the mass conservation and employing a modified form of Bernoulli's law for the coupling of the velocity-pressure corrections. When a potential velocity is introduced, where the velocity correction is applied in order to fulfil continuity equation, an additional equation for the potential of the velocity is introduced. The boundary conditions (BCs) for the velocity-correction potential function require the computation of the unit normal vectors. It is usually a difficult task, especially in the case of irregular geometries. In the proposed scheme there is no need to compute a potential velocity and unit normal vectors.

Both FDM and FVM methods widely use a semi-staggered or a fully staggered grid, applied in flow problems with uniform spatial domain or with some kind of symmetry. Although the applicability of the method in irregular geometries is feasible, the computational cost may increase drastically. On the other hand, mesh-based methods, despite their success, have some serious drawbacks related to the mesh generation. Mesh generation is still a difficult task, especially for 3D geometries, being the bottleneck of the entire simulation procedure. The main drawback is the refinement process. Eventually, meshless methods have recently emerged as a possible alternative to overcome the problems of mesh generation and facilitate local refinement of the approximation scheme.

In the context of Meshless methods (MM) the spatial domain is represented by a set of nodes, uniformly or randomly distributed along the interior and on the boundaries, without any inter-connectivity. A practical overview of meshless methods based on global weak forms was given in [15]. Numerous MMs schemes were developed in both Eulerian and Lagrangian frameworks, such as the Meshless Local Petrov-Galerkin (MLPG) [16–21], Local Boundary Integral Equation (LBIE) [22,23], Meshless Point Collocation (MPC) [24–28], Element Free Galerkin (EFG) [29,30], Smoothed Particle Hydrodynamics (SPH) [31–33] and Finite Point method [34–36], applied on the numerical solution of (N-S) equations. Flow equations can be solved in their primitive variables formulation or in their velocity-vorticity and stream function-vorticity formulation. In most of these methods pressure has been computed explicitly or as a final outcome, given the boundary conditions for pressure.

The present study deals with the reformulation of the implicit potential (IPOT) methodology [14] and its application in the context of meshless methods. The proposed scheme solves numerically the steady state, laminar and incompressible (N-S) equations, in their primitive variables formulation using a collocated or semi-staggered nodal arrangement. The novelty relies on the introduction of a complementary pressure (pressure correction) through the introduction of a complementary velocity, which ensures mass conservation. Moreover, we assume that the complementary velocity and pressure correspond to a complementary flow. Consequently, an "appropriate" momentum equation appears, which can be described as a modified expression of Bernoulli's law for the complementary flow and, after some algebra, the complementary pressure is obtained. In fact, both pressures, complementary and physical, are calculated through an algebraic relation without solving any partial differential equation. Eventually, the number of equations solved decreases. To the authors' knowledge, this is the first attempt to apply the proposed IPOT methodology using meshless schemes in general and, specifically, the MMLS method, to approximate the flow variables. The proposed IPOT meshless point collocation (MPC) scheme has the following characteristics: (*i*) it can be applied, in a straightforward manner, to steady, unsteady, internal and external fluid flows in 2D and 3D, *(ii)* it is equally performant for regular an irregular geometries, *(iii)* a distribution of points is sufficient, no numerical integration in space nor any mesh structure is required, *(iv)* there is no need for pressure boundary conditions since no pressure constitutive equation is solved, *(v)* it is quite simple and accurate, *(vi)* results can be obtained using collocated or semi-staggered nodal distributions, (*vii*) there is no need neither for the computation of the velocity potential nor the computation of the unit normal vectors and *(viii)* there is no need for a curvilinear system of coordinates.

The rest of the paper is organized as follows. In Section 2, the governing equations along with the proposed IPOT numerical method are presented, while the approximation method of the classical Moving Least Squares (MLS) and the Modified MLS are briefly presented in Section 3. Section 4 presents the verification benchmark flow problems used along with the test cases used to demonstrate and highlight the accuracy, robustness, and computational efficiency of the proposed scheme. Finally, the conclusions are given in Section 5.

2. Governing equations and solution procedure

2.1. Governing equations

Navier-Stokes equations express conservation of linear momentum. They are a set of nonlinear partial differential equations (PDEs) which, in velocity-pressure formulation [2], can be written in non-dimensional form as:• Momentum equation

$$(\boldsymbol{u}\bullet\nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2\boldsymbol{u} + \boldsymbol{F}, \tag{1}$$

• Continuity equation

$$\nabla\bullet\boldsymbol{u} = 0. \tag{2}$$

where $\boldsymbol{u}$ is the flow velocity vector, p is the pressure field, Re is the Reynolds number and $\boldsymbol{F}$ corresponds to body force terms (herein we assume $\boldsymbol{F}=\boldsymbol{0}$). All field variables are functions of space $\boldsymbol{x}$, in a fixed domain Ω surrounded by a closed boundary. The system of PDEs (1)-(2) is closed with appropriate boundary conditions related to the physical problem considered. Different types of BCs can be used, namely Dirichlet, Neumann, Robin, mixed type etc. In the present paper the applied boundary conditions are described in the numerical examples examined.

2.2. Solution procedure with IPOT scheme

In the context of the strong form meshless point collocation method, an iterative scheme has been developed for the numerical solution of the (N-S) equations in their primitive variables (velocity-