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Single layer regularized meshless method for three dimensional exterior acoustic problem

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ABSTRACT

The Regularized Meshless Method (RMM) is a meshless boundary method. Its source points and physical points are overlapped. The substraction and adding-back technique is utilized to avoid the singularity of the fundamental solution. It is simple and easy to be programmed. But the double layer potential should be adopted in the desingularity technique. Here the single layer potential is employed to circumvent the singularity. The substraction and adding-back technique is succeeded, but the careful selection of particular solution for the null-fields boundary integral equation is chosen to derive the diagonal elements for the Laplace Dirichlet problem. By this particular solution, the diagonal elements can be represented by the single layer potential. Here it is extended to the exterior Helmholtz problem by relationships between Laplace and Helmholtz singularities. The fictitious frequencies are avoided by the Burton-Miller type formula and Dual Surface technique. The accuracy of these methods are shown by three typical examples.

1. Introduction

The Method of Fundamental Solutions (MFS) is a typical meshless boundary collocation method. However the choice of source points is arbitrary and without a particular rule. Many Boundary Meshless Methods with source points coincident with physical points have been proposed in the literature. These methods use different techniques to avoid the singularity of fundamental solution. Boundary Node Method (BNM) [1] adopts the interpolation procedure to circumvent the singularity. And it was extended to 2-D interior Helmholtz problem [2]. Boundary Points Method (BPM) [3] uses the 'moving elements' to avoid the singularity. Boundary Particle Method (BPM) [4], Boundary Knot Method (BKM) [5] employ an alternative non-singular kernel function to circumvent the singularity. Boundary Distributed Source (BDS) method [6], Improved Boundary Distributed Source (IBDS) [7], Non-Singular Method of fundamental solution [8] remove the singularities by distributed source over areas (for 2D) or volumes (for 3D) covering the source points.

Regularized Meshless Method (RMM) which uses the desingularization of subtracting and adding back technique was proposed by Young et al. [9] for 2-D Laplace problem, and then applied to different problems [10–13]. This method was later extended to 2-D [14] and 3-D [15] exterior acoustic problem. In RMM the double layer potential was adopted as the fundamental solution for the convenience of using null-fields boundary integral equation to desingularize the fundamental solution for Laplace equation. Then Helmholtz equation fundamental solution is represented by its direct relation with Laplace equation. In this paper the substraction and adding-back technique is succeeded, but the careful selection of particular solution for the null-fields boundary integral equation is chosen to derive the diagonal elements for the Laplace Dirichlet problem. By this particular solution, the diagonal elements can be represented by the single layer potential [16]. Here it is extended to Helmholtz problem.

This paper is also similar to the idea of Singular Boundary Method [17,18] to get the magnitude of singular source, which also uses the single layer potential as the fundamental solution. This method also extended to 2-D interior [19] and exterior [20] acoustic problem. In the SBM [17] or ISBM [18], the inverse interpolation technique (IIT) was adopted to get the singular source magnitude for the Dirichlet problem. They got the Neumann problem singular source magnitude by the subtraction and adding-back technique in null-fields boundary integral equation firstly, then integrated the solution to achieve the Dirichlet problem singular source magnitude, the constant for the integration was derived by the inverse interpolation from the domain points. In this paper the Dirichlet problem singular source magnitude is directly derived from the null-field integral equation by the subtraction and adding-back technique using the single layer potential, without the needing of inverse interpolation. Recently the explicit empirical formula for the diagonal elements has been proposed [21]. It will be compared in numerical example 5.1.

In the following sections, the theory of the Single Layer Regularized Meshless Method (SRMM) for exterior acoustic problem is introduced. The Burton-Miller technique and Dual Surface technique are adopted to avoid the non-uniqueness. Then three typical examples show the validation of these methods.

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2. Formulation of single layer regularized meshless method

From the boundary value problem Helmholtz equation in 3D domain Ω exterior to a closely boundary Γ

$$\nabla^2 p(x) + k^2 p(x) = 0, x \in \Omega \tag{1}$$

subjected to the following boundary conditions

$$p(x) = \overline{p}(x), \quad x \in \Gamma_D \text{ (Dirichlet boundary condition)} \tag{2}$$

$$q(x) = \frac{\partial p}{\partial \boldsymbol{n}}(x) = \overline{q}(x), \quad x \in \Gamma_N \text{ (Neumann boundary condition)} \tag{3}$$

where Ω is a bounded domain with boundary $\Gamma = \Gamma_D + \Gamma_N$, $\boldsymbol{n}$ presents the outward normal, $k = \omega/c$ is the wavenumber, ω is the angular frequency, c is the wave speed in the medium Ω, p is the complex valued amplitude of radiated and/or scattered wave.

$$p = \begin{cases} p_R = p_T, & \text{if radiation} \\ p_S = p_T - p_I, & \text{if scattering}, \\ p_{R+S} = p_T - p_I, & \text{if both} \end{cases} \tag{4}$$

where the subscripts T, R and I denote the total, radiation and incidence wave respectively.

By the single layer fundamental solutions, the approximate solutions $p(x)$ and $q(x)$ of exterior acoustic problem can be expressed as follows:

$$p(x_i) = \begin{cases} \sum_{j=1}^{N} \alpha_j G(x_i, s_j), & x_i \in \Omega \\ \sum_{j=1, j\neq i}^{N} \alpha_j G(x_i, s_j) + \alpha_i G(x_i, s_i), & x_i \in \Gamma \end{cases} \tag{5}$$

$$q(x_i) = \frac{\partial p(x_i)}{\partial n_{x_i}} = \begin{cases} \sum_{j=1}^{N} \alpha_j \dfrac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i}}, & x_i \in \Omega \\ \sum_{j=1, j\neq i}^{N} \alpha_j \dfrac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i}} + \alpha_i \dfrac{\partial G(x_i, s_i)}{\partial \boldsymbol{n}_{x_i}}, & x_i \in \Gamma \end{cases} \tag{6}$$

where x_i is the i-th physical point, s_j is the j-th source point located on the physical boundary, α_j the j-th unknown intensity of the distributed source at s_j, N the numbers of source points and

$$G(x_i, s_j) = \frac{e^{ikr}}{r} \tag{7}$$

$$\frac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i}} = \frac{e^{ikr}}{r^3} \langle (x_i - s_j), \boldsymbol{n}_{x_i} \rangle (ikr - 1) \tag{8}$$

is the fundamental solution and the physical normal derivative of three-dimensional Helmholtz equation, $r = \| x_i - s_j \|$, $\langle , \rangle$ denotes the inner product. If the collocation points and source points coincide, the singularities are encountered. However, unlike the potential problem, the source intensity factors can't be calculated directly from the Helmholtz fundamental solutions by analytical-numerical technique. Fortunately, the Helmholtz fundamental solutions have a similar order of singularities as the related Laplace fundamental solutions. Hence the corresponding relationships can be represented by the following asymptotic expressions [22]

$$G(x_i, s_i) = G_L(x_i, s_i) + ik, \quad x_i \to s_i \tag{9}$$

$$\frac{\partial G(x_i, s_i)}{\partial \boldsymbol{n}_{x_i}} = \frac{\partial G_L(x_i, s_i)}{\partial \boldsymbol{n}_{x_i}}, \quad x_i \to s_i \tag{10}$$

$$\frac{\partial G(x_i, s_i)}{\partial \boldsymbol{n}_{s_i}} = \frac{\partial G_L(x_i, s_i)}{\partial \boldsymbol{n}_{s_i}}, \quad x_i \to s_i \tag{11}$$

$$\frac{\partial^2 G(x_i, s_i)}{\partial \boldsymbol{n}_{x_i} \partial \boldsymbol{n}_{s_i}} = \frac{\partial^2 G_L(x_i, s_i)}{\partial \boldsymbol{n}_{x_i} \partial \boldsymbol{n}_{s_i}} + \frac{k^2}{2} G(x_i, s_i), \quad x_i \to s_i \tag{12}$$

where $G_L(x_i, s_j)$ is the fundamental solutions of Laplace equation, $G_L(x_i, s_j) = 1/r$ in 3D problems.

The derivation of $G_L(x_i, s_i)$ and $\frac{\partial G_L(x_i, s_i)}{\partial \boldsymbol{n}_{x_i}}$ are shown in Appendix A. The formulas of $\frac{\partial G_L(x_i, s_i)}{\partial \boldsymbol{n}_{s_i}}$ and $\frac{\partial^2 G_L(x_i, s_i)}{\partial \boldsymbol{n}_{x_i} \partial \boldsymbol{n}_{s_i}}$ have been derived in [23].

As well already known that Eqs. (5), (6) encounters the non-uniqueness problems when the wave number k is the eigen-frequency of the corresponding interior problem. Many techniques to avoid the non-uniqueness exist in the literature. These techniques can be classified into two categories. One is to add additional restriction to get the unique solution, such as the CHIEF method [24] to add more points in the domain, which also satisfy the Helmholtz equation; The other one is to add the damping in the original equation, and shift the fictitious eigen-frequencies to the complex plane [25]. Many methods can be considered as this category. Burton-Miller Method [26] adds the imaginary double layer integral equation to the original one. Dual-surface method was original utilized in the electromagnetic scattering problem [27], and then extended to the acoustic scattering problem [28], which adapts the imaginary surface to shift the fictitious frequencies. Here we adapt these two damping methods with the regularized meshless method to overcome the non-uniqueness.

3. Burton-miller type regularized meshless method

Directly from the Burton-Miller concept, we can construct Burton-Miller type regularized meshless method.

$$p(x_i) = \begin{cases} \sum_{j=1}^{N} \alpha_j \left(G(x_i, s_j) + \gamma^{BM} \dfrac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{s_j}} \right), & x_i \in \Omega \\ \sum_{j=1, j\neq i}^{N} \alpha_j \left(G(x_i, s_j) + \gamma^{BM} \dfrac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{s_j}} \right) \\ + \alpha_i \left(G(x_i, s_i) + \gamma^{BM} \dfrac{\partial G(x_i, s_i)}{\partial \boldsymbol{n}_{s_i}} \right), & x_i \in \Gamma \end{cases} \tag{13}$$

$$q(x_i) = \frac{\partial p(x_i)}{\partial n_{x_i}} = \begin{cases} \sum_{j=1}^{N} \alpha_j \left(\dfrac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i}} + \gamma^{BM} \dfrac{\partial^2 G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i} \partial \boldsymbol{n}_{s_j}} \right), & x_i \in \Omega \\ \sum_{j=1, j\neq i}^{N} \alpha_j \left(\dfrac{\partial G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i}} + \gamma^{BM} \dfrac{\partial^2 G(x_i, s_j)}{\partial \boldsymbol{n}_{x_i} \partial \boldsymbol{n}_{s_j}} \right) \\ + \alpha_i \left(\dfrac{\partial G(x_i, s_i)}{\partial \boldsymbol{n}_{x_i}} + \gamma^{BM} \dfrac{\partial^2 G(x_i, s_i)}{\partial \boldsymbol{n}_{x_i} \partial \boldsymbol{n}_{s_i}} \right), & x_i \in \Gamma \end{cases} \tag{14}$$

Where $\gamma^{BM} = i/k$ according to [25].

4. Dual-surface single-layer regularized meshless method

The basic idea is to generate a virtual second surface inside the structure (Fig. 1) by shifting the original points s_j along the element normal to a “virtual” surface s_j^{DS} using a distance δ^{DS} which depends on the wavelength.

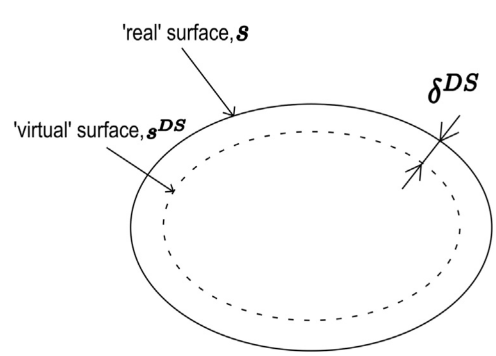

Fig. 1. Dual-surface model scheme.