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A linear complementarity formulation of meshfree method for elastoplastic analysis of gradient-dependent plasticity

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ABSTRACT

This work presents a linear complementarity formulation for elastoplastic analysis of the gradient-dependent plasticity including large deformation problems. The formulation is based on the meshfree smoothed radial point interpolation method, where the parametric variational principle (PVP) is used in the form of linear complementarity and the gradient-dependent plasticity is represented by the linearization of yield criterion. The yield stress is linearly evolved through equivalent plastic strain as well as its Laplacian (namely second gradient). The global discretized system equations are transformed into a standard linear complementarity problem (LCP), which can be solved readily using the Lemke method. The proposed approach is capable of simulating material hardening/softening and strain localization. An extensive numerical study is performed to validate the proposed method and to investigate the effects of various parameters. The numerical results demonstrate that the proposed approach is accurate and stable for the elastoplastic analysis of 2D solids with gradient-dependent plasticity on strain localization.

1. Introduction

The problem of strain localization is commonly observed in many materials such as concrete [1], rock [2] and soil [3] in the form of shear band, which is characterized by a deformation concentration in a narrow band. Due to the complexity of the problem, only a few simple cases may be solved analytically on the basis of strain localization condition [1], Cosserat media [4], nonlocal and gradient theories of plasticity [5,41] and damage [6]. For the problems of complex boundaries, numerical methods are needed because of the strong nonlinearity of the problem.

Many numerical schemes have been developed for solving strain localization problems. To summarize briefly, these include: 1) Finite element method (FEM) using an element-enrichment technique, with the strain localization condition identified by means of a bifurcation analysis [7]; 2) FEM based on the consideration of Cosserat continua [2,8,9]; 3) Methods based on nonlocal plasticity theory, such as regularization of integral using FEM [10], implicit gradient plasticity using FEM [11] or element-free Galerkin method (EFG) [12], explicit gradient plasticity (also called gradient-dependent plasticity) using FEM [13], boundary element method (BEM) [14] and EFG [12]; 4) in the environment of meshfree cohesive model/interface for shear band and cracking [38–40]; and 5) Other methods [15–21,41].

It is generally recognized that the methods based on Cosserat media and nonlocal theories of plasticity can provide a stable prediction of the width of the shear band. These methods can overcome the pathological phenomenon of mesh sensitivity for softening media because of the inclusion of an internal length scale parameter. However, methods based on Cosserat media can only be suited for problems which are dominated by a shear failure. In contrast, methods based on nonlocal theory may be applied to problems dominated by both shear and tension failure. Moreover, the gradient-dependent plasticity underlying these methods is easy to be implemented, and for this reason, these methods have attracted considerable attention in the research community. In this paper, the gradient-dependent plasticity is considered for strain localization problems.

For problems of strain localization, mesh distortion often occurs because of large deformation in a local zone or a shear band in the finite element frame, so a mesh adaptive strategy is often required to get rid of that problem. As an alternative, many meshfree/meshless methods [12,22–29] have been developed in recent years. Recently, a novel smoothed point interpolation method (*s*-PIM) has been developed based on the newly developed G space theory and weakened weak formulation [30]. With the help of the generalized gradient smoothing operation, S-PIM

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can effectively softened the overly-stiff stiffness and process a number of good properties, including better stress results, higher convergence rate and efficiency, immune from volumetric locking and providing upper bound energy solutions [31]. In this work, we further implement the node-based smoothed radial point interpolation method (NS-RPIM) for the elastoplastic analysis of 2D solids with gradient-dependent plasticity. The NS-RPIM is a typical model of S-PIM and was originally named as linearly conforming radial point interpolation method [30,32–34]. The NS-RPIM has the following particular advantages: 1) the shape function has the Kronecker delta property; 2) the moment matrix used for constructing shape functions is always invertible for irregular nodes; 3) the linear field can be exactly reproduced using RPIM shape functions augmented with linear polynomials; and 4) the method can exactly pass the standard linear patch test, which is very stable, accurate and efficient.

This paper is organized as follows. Section 2 provides an overview of the weak form of the boundary value problem in two dimensions and the gradient-dependent plasticity, as well as the linearization of typically used constitutive models. Section 3 gives the detailed formulations for elastoplastic analysis based on the NS-RPIM, which includes shape function generation, the generalized gradient smoothing operation, strong form of the problem, discrete governing equations and solution procedure. Three typical numerical examples are studied in Section 4 to validate the proposed approach and investigate the effect of associated parameters. Finally some conclusions are drawn in Section 5.

2. Fundamentals of 2D elastoplastic problem with gradient-dependent plasticity

In this section, the weak form of boundary value equations for elastoplastic analysis of 2D solid mechanics problems with gradient-dependent plasticity is briefly introduced. Then the gradient-dependent plasticity is reviewed, along with the linearization in terms of the equivalent plastic strain and the plastic multiplier in the incremental form. Finally two commonly used yield criteria and the corresponding flow rules, as well as their linearized forms, are described.

2.1. Weak form of boundary value equations in the incremental form

The static equilibrium equation governing the solid defined in domain Ω can be expressed as

$$\mathbf{B}^T\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \text{ in } \Omega \tag{2.1}$$

where $\boldsymbol{\sigma} = \left[\sigma_{xx}, \sigma_{yy}, \tau_{xy}\right]^T$ is a vector of stress increment, $\mathbf{b}$ represents the increment of body force density, and $\boldsymbol{B}$ is a partial derivative operator.

At an arbitrary point the geometric continuity equation (strain-displacement relation) can be written in the incremental form as

$$\boldsymbol{\varepsilon} = \mathbf{B}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\mathbf{u} \tag{2.2}$$

where $\boldsymbol{\varepsilon} = \left[\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}\right]^T$ is the incremental strain vector, and $\mathbf{u} = \left[u, v\right]^T$ represents the vector of incremental displacement components.

The elastoplastic constitutive equation with gradient-dependent plasticity at a random point is given as

$$f^y\left(\tilde{\boldsymbol{\sigma}}, \tilde{\bar{\varepsilon}}^p, \nabla^2\tilde{\bar{\varepsilon}}^p\right) = f(\tilde{\boldsymbol{\sigma}}) - f\left(\tilde{\bar{\varepsilon}}^p, \nabla^2\tilde{\bar{\varepsilon}}^p\right) \le 0 \tag{2.3}$$

$$\boldsymbol{\varepsilon}^p = \left(\partial f^g(\tilde{\boldsymbol{\sigma}})/\partial\boldsymbol{\sigma}\right)^T\delta = \bar{\mathbf{M}}^f\delta \tag{2.4}$$

$$\boldsymbol{\sigma} = \mathbf{D}\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\right) = \mathbf{D}\boldsymbol{\varepsilon} - \mathbf{D}\bar{\mathbf{M}}^f\delta \tag{2.5}$$

$$f\delta = 0 \quad (\delta \ge 0) \tag{2.6}$$

where the total stress $\tilde{\boldsymbol{\sigma}}$ is composed of the initial stress $\boldsymbol{\sigma}_t$ at time t and incremental stress $\boldsymbol{\sigma}$, $f(\tilde{\boldsymbol{\sigma}})$ and $f\left(\tilde{\bar{\varepsilon}}^p, \nabla^2\tilde{\bar{\varepsilon}}^p\right)$ are yield function and yield strength at the stress space, respectively, $\tilde{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3}\tilde{\varepsilon}^{p'T}\tilde{\varepsilon}^{p'}}$ is the equivalent plastic strain and $\tilde{\varepsilon}^{p'}$ is the deviatoric plastic strain, $\bar{\mathbf{M}}^f = \left(\partial f^g(\tilde{\boldsymbol{\sigma}})/\partial\boldsymbol{\sigma}\right)^T$ represents the plastic flow direction and f^g is the plastic potential function, $\boldsymbol{\delta}$ is the incremental plastic multiplier, and $\mathbf{D}$ is the elastic stiffness matrix.

The displacement boundary condition can be expressed as

$$\mathbf{u} = \bar{\mathbf{u}} \text{ on } \Gamma^u \tag{2.7}$$

where $\bar{\mathbf{u}}$ is a specified incremental displacement on boundary Γ^u.

The traction boundary condition is given as

$$\mathbf{T} = \mathbf{B}^T\left(n_x, n_y\right)\boldsymbol{\sigma} = \bar{\mathbf{T}} \text{ on } \Gamma^t \tag{2.8}$$

in which $\bar{\mathbf{T}}$ is a given incremental traction on boundary Γ^t, and $\mathbf{n} = \left[n_x, n_y\right]$ is the outward normal vector of the traction boundary.

2.2. Gradient-dependent plasticity

In the gradient-dependent plasticity [5], the yield strength depends not only on the hardening internal variable, but also on its Laplacian. It may be generally given in the form of an equivalent plastic strain $\tilde{\bar{\varepsilon}}$ as an internal variable as

$$f\left(\tilde{\bar{\varepsilon}}^p, \nabla^2\tilde{\bar{\varepsilon}}^p\right) = \sigma_0^y + h\tilde{\bar{\varepsilon}}^p + g\nabla^2\tilde{\bar{\varepsilon}}^p \tag{2.9}$$

where $h = \partial f\left(\tilde{\bar{\varepsilon}}^p, \nabla^2\tilde{\bar{\varepsilon}}^p\right)/\partial\tilde{\bar{\varepsilon}}^p$ is the hardening/softening modulus and $g = \partial f\left(\tilde{\bar{\varepsilon}}^p, \nabla^2\tilde{\bar{\varepsilon}}^p\right)/\partial\left(\nabla^2\tilde{\bar{\varepsilon}}^p\right)$ represents the gradient influence factor. For simplicity, constant h and g are adopted in this paper.

Considering that $\tilde{\bar{\boldsymbol{\varepsilon}}}^p = \bar{\boldsymbol{\varepsilon}}_t^p + \bar{\boldsymbol{\varepsilon}}^p$, the linearization form of the yield strength in terms of the incremental plastic strain, $\bar{\varepsilon}^p$, can be obtained as

$$f\left(\bar{\varepsilon}^p, \nabla^2\bar{\varepsilon}^p\right) = \sigma_t^y + h\bar{\varepsilon}^p + g\nabla^2\bar{\varepsilon}^p \tag{2.10}$$

in which $\sigma_t^y = \sigma_0^y + h\bar{\varepsilon}_t^p + g\nabla^2\bar{\varepsilon}_t^p$.

Furthermore, the incremental equivalent plastic strain may be expressed using the incremental plastic multiplier as

$$\bar{\varepsilon}^p = \alpha_t\delta \tag{2.11}$$

where $\alpha_t = \frac{2}{3}\mathbf{M}^{ps}\mathbf{M}^f$, $\mathbf{M}^{ps} = \left[\left(1 + c_{ps}\right)\varepsilon_{xx}^p\left(1 + c_{ps}\right)\varepsilon_{yy}^p\varepsilon_{xy}^p\right]/\bar{\varepsilon}_t^p$, $c_{ps} = \left(2\alpha_{ps}^2 - 2\alpha_{ps} - 1.0\right)/3.0$, and α_{ps} is a coefficient with a value of 0 for plane stress problem and $\nu/(1-\nu)$ for plane strain problem [5].

Then the Laplacian of the equivalent plastic strain can be expressed as

$$\nabla^2\bar{\varepsilon}^p = \nabla^2\alpha_t\delta + 2\left(\nabla\alpha_t\right)^T\nabla\delta + \alpha_t\nabla^2\delta \tag{2.12}$$

Substituting Eqs. (2.11) and (2.12) into (2.10), the linearized yield strength in terms of the incremental plastic multiplier is obtained as

$$f\left(\bar{\varepsilon}^p, \nabla^2\bar{\varepsilon}^p\right) = \sigma_t^y + \bar{h}\delta + \bar{\mathbf{g}}_1\nabla\delta + \bar{g}_2\nabla^2\delta \tag{2.13}$$

where $\bar{h} = h\alpha_t + g\nabla^2\alpha_t$, $\bar{\mathbf{g}}_1 = 2g\nabla^T\alpha_t$, and $\bar{g}_2 = \alpha_t g$. Note that by comparing with Eq. (2.10), the above expression contains an