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Using the Gaussian function to simulate constant potential anodes in multiobjective optimization of cathodic protection systems

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ABSTRACT

The purpose of this work is to numerically find the optimum location of constant potential anodes to ensure complete structure surface protection using a cathodic protection technique. The existence of sacrificial anodes is originally introduced through the boundary conditions of the corresponding boundary value problem (BVP). However, if constant potential galvanic regions are introduced through its boundaries, then finding their optimal location is not an easy task due to the necessity of redefining boundary geometric nodes and the arrangement of virtual sources for the standard method of fundamental solutions (MFS) formulation. Therefore, in this work, the galvanic anodes are introduced as source terms using a Gaussian function. Hence, the boundary remains the same for different anode positions. The optimization process includes the identification of the following parameters characterizing the Gaussian function: the optimum coordinates of the centre of the anode, a factor that involves the inherent potential of the electrode and a proportionality factor for the electrode diameter. The MFS methodology coupled with a genetic algorithm presented good results for this multiobjective optimization procedure. This fact can be seen in the several results of applications that are discussed in this paper, considering numerical simulations in finite regions in R^2.

1. Introduction

Cathodic protection (CP) is a technique to reduce the corrosion rate of the metal surface by making it the cathode of an electrochemical cell. In practice, the goal is to provide a uniform potential distribution on the metal surfaces, limited by a minimum potential value to guarantee protection from the corrosion using an external power source (impressed current CP) or by utilizing a sacrificial anode [1,2]. In this work, it is considered the case of sacrificial anode system, where the galvanic relationship between a sacrificial anode material, such as zinc or magnesium, and the structure is used to supply the required CP current.

The most commonly used methods for modelling cathodic protection systems are finite element method (FEM) and boundary element method (BEM). However, the method of fundamental solutions (MFS) is a technique which can also be applied to CP problems (see references [3,4]). Just like BEM, MFS is applicable when a fundamental solution of the differential equation in question is known, with the advantage of not requiring any integration procedure or specific treatment for the singularities of the fundamental solution. The BEM and the MFS are the most appropriate techniques to solve problems involving galvanic corrosion and CP systems, mainly to solve large problems and considering homogeneous conductive medium. These methods require only the representation of anodes and cathodes surfaces, which leads to better resolution and reduction in computer run time when compared to FEM. Mathematical simulations of cathodic protections systems using FEM can be seen in [5–7], where compact support functions were used to simulate constant potential electrodes in electrochemical process.

Due to its accuracy and simplicity of mesh generation, the BEM is usually used for numerical simulations of sacrificial anode CP systems. Miyasaka et al. [8], for example, evaluated the computational accuracy of BEM to estimate the galvanic corrosion and CP in an actual field. Abootalebi et al. [9] determined the optimum location of zinc anode electrode using BEM in 2D. In addition, the influence of anode length and paint defect on corrosion current density and potential distributions of sacrificial anode CP system were investigated. Several different applications of BEM to study CP systems have been reported in the literature, including reference to practical analyses performed by offshore oil companies [10–12]. The BEM implementation includes a Newton–Raphson solution algorithm to accommodate possible nonlinear boundary conditions [13]. Coupled with the numerical method to solve the Laplace equation, optimization algorithms can be used to

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determine the optimum location and the corresponding current intensity values of the anodes in order to satisfy a protection criterion. Hence, the minimization of an objective function using, for example, genetic algorithms (GAs) and a penalty method for handling constraints can be adopted. This type of optimization can and has also been successfully performed using BEM [14,15].

Kupradze and Aleksidze [16] first proposed the basic ideas for the formulation of the MFS. In order to construct the solution, the MFS uses only a superposition of fundamental solutions associated to the problem, with singular points (virtual sources) located outside the domain. The accuracy of the MFS numerical solution depends on the radius of such a circle or on the distance from the virtual sources over the geometrically similar boundary contour to the problem boundary, especially due to possible ill-conditioning and/or rank-deficiency of the algebraic system of equations formed. The MFS has successfully been applied for solving several problems. For example, Costa et al. [17] developed numerical frequency domain formulations to simulate the 2D acoustic wave propagation in the vicinity of an underwater configuration which combines two sub-regions using the MFS. More recently, Fontes Jr. et al. [18] applied a regularized method of fundamental solutions coupled with the numerical Green's function procedure to solve linear elastic fracture mechanics (LEFM) problems. Problems with nonlinear boundary conditions solved by MFS can be treated as nonlinear least squares problems [19]. Therefore, to determine the coefficients of the linear superposition of the fundamental solutions and the positions of the virtual sources, a nonlinear least squares algorithm is found necessary. In the present work, the minimization of the nonlinear functional is done using the MINPACK [20] routine LMDIF, which is a modified version of the Levenberg–Marquardt algorithm [21]. The Levenberg–Marquardt method has been highly recommended when Jacobian is rank-deficiency or nearly so.

Santos et al. [3] were the first to use standard MFS successfully in the numerical simulations of CP systems. In the paper cited, the authors proposed a GA with the MFS to simulate cathodic protection systems with nonlinear boundary conditions. The adopted GA was used to minimize a nonlinear error function, whose design variables were the coefficients of the linear superposition of fundamental solutions and the positions of the virtual sources, randomly distributed outside the problem domain. In Santos et al. [4] was presented a formulation using a GA and the MFS to determine the optimum location and the optimum current intensity of the anodes inserted in the electrolyte leading to a practical optimized design procedure. The results presented in this paper included a comparison with a direct boundary element (BEM) solution procedure.

The purpose of the present paper is to use a genetic algorithm (GA) with the MFS for optimizing the anode position in a galvanic cathodic protection system. Furthermore, the constant potential circular anodes are here mathematically represented by a Gaussian function. Thus, the GA will be used to search the Gaussian function parameters: coordinates of the centre of the anode, a factor that involves the inherent potential of the anode and a proportionality factor for the anode diameter. The main advantage of considering constant potential regions as the source term is that the boundary conditions remain the same, no matter where the anodes are located. The circular anodes are considered and, therefore, a good approximation for the source term is a function with circular compact support like the Gaussian function. In the MFS case, it is not necessary for the arrangement of the virtual sources inside the galvanic anodes (cavities), when they are introduced as source terms. A particular solution to the inhomogeneous differential equation resulting will be evaluated using the dual reciprocity method (DRM). In the DRM, the source term is approximated by a finite series of radial basis functions with an approximation to particular solution calculated analytically from source [22]. The corresponding homogeneous solution is found by MFS.

This text is organized as follows: in Section 2 is presented a standard electrochemical potential problem, i.e., the Laplace equation with boundary conditions given by constant potential (anode) and polarization curve (cathode). The decision of introducing the Gaussian function to model constant potential galvanic regions is also discussed in Section 2. The standard MFS formulation with the dual reciprocity method for solving the Poisson equation with nonlinear boundary conditions is discussed in Section 3. The multiobjective optimization problem and the genetic algorithm are showed in Section 4. This algorithm is coupled with MFS to guarantee protection from the corrosion by utilizing a sacrificial anode in the numerical examples solved in Section 5. The paper ends with some discussions found in Section 6.

2. Boundary value problem (BVP)

For a homogeneous and isotropic electrolyte (domain Ω) system of conductivity k, as illustrated in Fig. 1, the electrochemical potential problem studied obeys the Laplace equation:

$$k\nabla^2\phi(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega, \tag{1}$$

subjected to the following boundary conditions:

$$i(\mathbf{x}) = F(\phi), \quad \mathbf{x} \in \Gamma_1, \tag{2}$$

$$\phi(\mathbf{x}) = \phi_0, \quad \mathbf{x} \in \Gamma_2, \tag{3}$$

where $\Gamma \equiv \Gamma_1 \cup \Gamma_2$ is the boundary of Ω, ϕ_0 is the fixed potential of anode, $i(\mathbf{x})$ is the current density in the outward normal direction $\mathbf{n}$ and $F(\phi)$ is a nonlinear function of ϕ given here by the following polarization curve [23]:

$$i = F(\phi) = e^{\frac{\phi+693.91}{\beta_1}} - \left[\frac{1}{i_1} + e^{\frac{\phi+521.6}{\beta_2}}\right]^{-1} - e^{-\frac{\phi+707.57}{\beta_3}}, \tag{4}$$

with ϕ and i having units mV and μA/cm^2, respectively, and β_1, β_2, β_3 and i_1 are given constant parameters: $\beta_1 = 24$ mV,

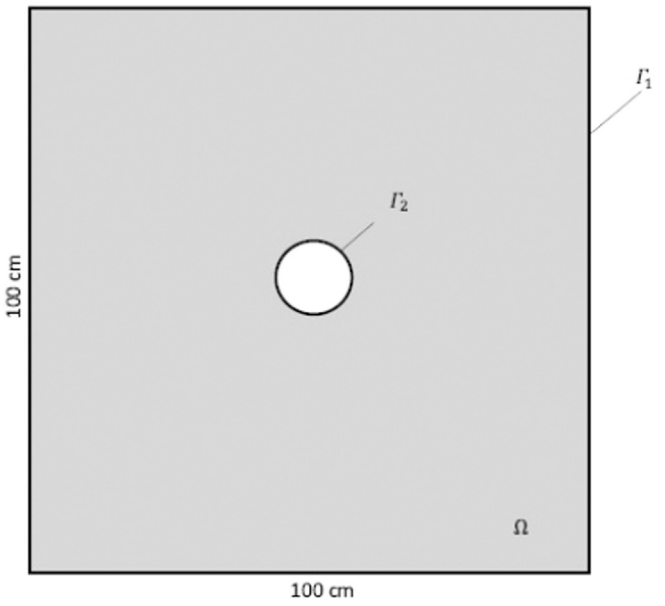

Fig. 1. The original potential problem.