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# A new dual reciprocity hybrid boundary node method based on Shepard and Taylor interpolation method and Chebyshev polynomials

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ABSTRACT

A new dual reciprocity hybrid boundary node method (DHBNM) is proposed in this paper, in which the Shepard and Taylor interpolation method (STIM) and Chebyshev polynomials interpolation are proposed. Firstly, the Shepard interpolation is used to construct zero level shape function, and the high-power shape functions are constructed through the Taylor expansion, and through those two methods, no inversion is needed in the whole process of the shape function construction. Besides, Chebyshev polynomials are used as the basis functions for particular solution interpolation instead of the conical function, radial basis functions, and the analytical solutions of the basic form of particular solutions related to Chebyshev polynomials for elasticity are obtained, by means of this method, no internal node is needed, and interpolation coefficients can be given as explicit functions, so no inversion is needed for particular solution interpolation, which costs a large amount of computational expense for the traditional method. Based on those two methods, a new dual reciprocity hybrid boundary node method is developed, compared to the traditional DHBNM, no inversion is needed for both shape function construction and particular solution interpolation, which greatly improves the computational efficiency, and no internal node is needed for particular solution interpolation. Numerical examples are given to illustrate that the present method is accurate and effective.



## 1. Introduction

In the past 30 years, meshless methods have been developed rapidly, and a lot of different meshless methods have been proposed one after another, and some of them have been widely used in practical engineering. At the same time, for constructing different meshless methods, a lot of shape function constructing methods for different meshless methods have been proposed, such as: the moving least square(MLS), the point interpolation method, the Kriging interpolation and so on [15]. As a widely used approximation method, MLS has been firstly proposed by Lancaster and Salkauskas [14], and then has been widely applied for different kinds of meshless methods, such as the element-free Galerkin method (EFG) [3], the meshless local Petrov-Galerkin method (MLPG) [2,11], the local boundary integral equation method (LBIM) [1,43], the boundary node method (BNM) [24] and so on. In those methods, although no element is needed for the variable interpolation, background elements are inevitable for 'energy' integration. Besides, some other methods such as mesh regeneration algorithm [12,39] and meshless singular boundary method have been developed recently.

In order to overcome the defect of background element for the boundary node method, applying hybrid displacement variational formulation and three fields interpolation scheme, Zhang and Yao [40,41] proposed the hybrid boundary node method(HBNM) and the regular hybrid boundary node method(RHBNM). Later, based on HBNM and rigid body displacement, Miao and Wang [19,20] developed a meshless method of singular hybrid boundary node method(SHBNM), later, they applied this method for analysis of reinforced concrete members and 3D composite materials [21–23]; and furthermore, applying dual reciprocity method (DRM) [32] into SHBNM, [36,37,38] proposed the dual reciprocity hybrid boundary node method(DHBNM) to solve inhomogeneous, dynamic, nonlinear problems, and so on.

The HBNM, RHBNM, SHBNM and DHBNM are MLS-based meshless methods. As an approximation method, the shape function based on MLS lacks the Delta function property compared with the widely used shape function obtained by interpolation, so boundary conditions cannot be imposed easily and directly, and its frequently inversion operation is inefficient. Aimed to those defects, the radial basis function interpolation method [16, 42], Kriging interpolation method [30], partition of unity [18] have been widely used to construct meshless shape function in past decades,

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and Cai and Zhu [5,6] have proposed meshless Shepard interpolation method, which satisfies the Delta function property and has high order completeness. Based on the Shepard interpolation method and Taylor expansion, [33] proposed the Shepard and Taylor interpolation method(STIM). As a shape function constructing method, the advantages of STIM are: the interpolation property, the arbitrarily high order consistency, no inversion for the whole process of shape function constructing, and the low computational expense.

To avoid domain integral that comes out from the inhomogeneous term of governing equation, the dual reciprocity method(DRM) was introduced by Nardini and Brebbia [26,27] in 1982 for elastodynamic problems and extended by Wrobel and Brebbia [32] to time-dependent diffusion in 1986. Later, a book by Partridge et al. [28] has been published to introduce dual reciprocity method to apply for boundary element method. In the first few years, the conical function $1 + r$ was exclusively employed for approximation of inhomogeneous term. After that the theory of radial basis function for DRM was introduced by Golberg and Chen [9] to replace the conical function. Since then, a lot of important papers about DRM have been focused on the investigation of the effect of choosing different radial basis functions [7,25].

Actually, a good choice of radial basis functions improves the accuracy and efficiency of DRM, and it is widely accepted as a reliable numerical method in transferring the domain integral to the boundary in the BEM community. But in those methods, the inversion for DRM processes is inevitable, which costs much large computational time, and it is inefficient for large scale calculation. It is well-known that Chebyshev polynomials are valuable tools in numerical analysis and approximation theory [17], and they are widely used in the numerical solution of boundary value problems for partial differential equations with spectral methods [4], which has a rapid convergence rate. Golberg et al. [10] used the symbol software mathematica to connect monomials with Chebyshev polynomials and employed their derived particular solution for floating number computing. Then Reutskiy and Chen [29] circumvented the tedious book keeping by using two-stage approximations of trigonometric functions and Chebyshev polynomials. Later, [13] used Chebyshev polynomials for approximating particular solutions of elliptic equations, and Tsai [31] took his effort for the particular solutions of Chebyshev polynomials for Reissner plates under arbitrary loadings.

In this paper, in order to overcome the inefficient property of the traditional dual reciprocity hybrid boundary node method (DHBNM), a new dual reciprocity hybrid boundary node method (DHBNM) is proposed, in which the Shepard and Taylor interpolation method(STIM) is employed for shape function constructing, and Chebyshev polynomials are applied for basis functions of particular solution interpolation. Firstly, the Shepard interpolation is used to construct zero level shape function, and the high-power shape functions are constructed through the Taylor expansion, and through those methods, STIM is developed, and no inversion is needed in the process of the shape function construction, and much lower computational expense is achieved. At the same time, Chebyshev polynomials are used as basis functions for particular solution interpolation instead of the conical function, radial basis functions, by means of this method no real internal interpolation node is needed, and the interpolation coefficients can be given as explicit functions, then no inversion is needed in the process of particular solution interpolation, which costs a large amount of computational expense for the traditional method. Based on those two methods and hybrid boundary node method, a new dual reciprocity hybrid boundary node method is developed, compared to the traditional DHBNM, no inversion is needed for both shape function construction and particular solution interpolation process, which greatly improve the computational efficiency.

## 2. Description of governing equation

In this paper, we take elasticity problem as the example, then consider an elasticity problem in domain $\Omega$ bounded by $\Gamma$. The governing equation can be given

$$\sigma_{ij,j} = b_i \tag{1}$$

$$u_i = \hat{u}_i \text{ on } \Gamma_u \tag{2}$$

$$t_i = \sigma_{ij} n_j = \hat{t}_i \text{ on } \Gamma_t \tag{3}$$

In which the superposed bar denotes the prescribed boundary values and $\mathbf{n}$ is the unit vector of outward normal of boundary, $b_i$ is the inhomogeneous term.

According to the traditional DHBNM theory, the solution variable of displacement $u$ can be divided into the complementary solution $u^c$ and the particular solution $u^p$, which can be expressed as [36–38]

$$u_i = u_i^c + u_i^p \tag{4}$$

The complementary solution $u_i^c$ must satisfy the homogeneous equation and the modified boundary conditions, but the particular solution just satisfies the inhomogeneous equation in the whole space.

The particular solution $u_i^p$ can be solved by Chebyshev polynomials interpolation in Section 4 by means of dual reciprocity method. The complementary solution $u^c$ must satisfy the homogeneous equation and the modified boundary conditions, according to modified variational principle of hybrid boundary node method, we can get the local integral of the present method,

$$\int_{\Gamma_s} (t - \tilde{t}) h_J(Q) d\Gamma - \int_{\Omega_s} \sigma_{ij,j} h_J(Q) d\Omega = 0 \tag{5}$$

$$\int_{\Gamma_s} (u - \tilde{u}) h_J(Q) d\Gamma = 0 \tag{6}$$

In which $\Gamma_s$ is the interaction of sub-domain $\Omega_s$ and the boundary of the calculation domain, which can be seen in references [36–38], and test function $h_J(Q)$ in the present method is given as

$$h_J(Q) = \begin{cases} \dfrac{\exp\left[-(d_J/c_J)^2\right] - \exp\left[-(r_J/c_J)^2\right]}{1 - \exp\left[-(r_J/c_J)^2\right]} & 0 \le d_J \le r_J \\ 0 & d_J \ge r_J \end{cases} \tag{7}$$

in which the variables, the factors and its related contents can be referred in [36–38].

## 3. Shepard and Taylor interpolation method

MLS is a widely used shape function construction method for meshless methods, and as an approximation method, MLS has high accuracy, but it has three disadvantages, firstly, it is lack of the Delta function property, so the boundary condition cannot be easily and directly imposed; secondly, high computational expense is needed, because individual interpolation coefficients are needed for every interpolation nodes; finally, the inversion is inevitable for every nodes in their shape function constructing processes. To overcome those defects, the Shepard and Taylor interpolation