Engineering Analysis with Boundary Elements 73 (2016) 95–102

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Flows in slip-patterned micro-channels using boundary element methods

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ARTICLE INFO

Article history:
Received 6 January 2016
Received in revised form
4 July 2016
Accepted 10 September 2016

Keywords:
Boundary element method
Slip patterning
Stokes equation

ABSTRACT

In this study we investigate steady, pressure-driven, two-dimensional flow of Newtonian fluid through slip-patterned, rectangular channels in the low Reynolds number limit. The slip flow regime is modeled using the Navier's slip boundary condition. In this work, we present only in-phase patterned slip. Subsequently, based on the characteristic length of the patterning, we have considered two subcases, namely large and fine patterned slip. Boundary element method (BEM) is used to numerically solve Stokes equation and obtain the streamline profiles. Streamlines, velocity profiles, pressure gradients, and shear stresses are analyzed to gain a proper understanding of the flow mechanics.
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1. Introduction

Modeling viscous incompressible flows over solid surfaces require significant assumptions about the boundary conditions at the solid–liquid interface. One of the most accepted boundary condition is the *no-slip* boundary condition which assumes that the velocity of the fluid element adjacent to a solid boundary is equal to the velocity of the solid boundary. This is an over-simplification of the effect of the solid wall on the fluid and is correct only in certain specific circumstances. Thin film dynamics, problems involving multiple interfaces, flows over hydrophobic surfaces, and flow of rarefied or rheological fluids are a few instances where the no-slip boundary condition fails. Over the years, in an attempt to correctly model the interaction between the solid wall and the liquid, various ingenious models have been developed.

Historically, Navier [1] was the first to propose a slip boundary condition on a solid wall. This model states that the slipping velocity of the fluid at the solid–liquid interface should be proportional to the shear stress at that point. Following this, Helmholtz and Pitrowski [2] introduced the concept of *slip-length* for the occurrence of slip next to a solid wall. Studies by Kundt and Warburg [3] suggested that the coefficient of slip was inversely proportional to the pressure. Later, Maxwell [4] proposed a model to describe the slip flow of a gas adjacent to a solid boundary. A thorough historical discussion of the prior work in this field can be found in Kennard [5]. Several experiments have been carried out to understand the fundamental nature of slip over solid surfaces. One of the early endeavors of calculating the slip at the wall was undertaken by Mooney [6] who introduced an approximate methodology to calculate the slip velocity (the Mooney technique). Early experiments done on capillary rheometers by Vinogradov and Ivanova [7] revealed that the slip velocity might also depend on the normal stresses. Rajagopal et al. [8] have considered the dependence of slip velocity on both shear stress and normal stress, and shown that in general traditional methods of calculating the slip velocities, like the Mooney Method, might be ineffective. Numerous studies aimed at analyzing the effect of other important factors, such as, viscous heating and molecular weight, on wall slip of molten polymers [9,10]. The slip length introduces a virtual surface, on which the velocity may be zero below the actual surface. Various factors influence the range of slip length. For example, a large slip length can be seen in situations where nanobubbles are trapped on a hydrophobic surface [4,6,5,7]. Irregular surfaces are encountered in most applications involving microchannels. Imposing such complex boundary conditions in analytical solutions might be a big challenge. Thus, one has to rely on computational methods.

Computational study of fluid flow problems is not a new domain of study. A variety of techniques such as finite difference method (FDM), finite element method (FEM) and finite volume method (FVM) have been used to investigate fluid flows over complex geometries. A significant advantages of boundary element method (BEM) over the techniques mentioned above is that (i) the dimension of the computational domain is reduced by order one, thus saving memory and computational time, and (ii) the solution can be achieved at any point inside the domain.

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http://dx.doi.org/10.1016/j.enganabound.2016.09.006
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Currently, the ability to fabricate structures and patterns on the micrometer and nanometer length scale [11,12] has triggered a wide range of scientific investigations. Slip or no-slip condition between a solid surface and a fluid strongly depends on the nature of interaction between them. Hydrophobic surfaces portray a 'slip' boundary condition whereas hydrophilic surfaces generally give rise to the no-slip condition. Various studies on chemical modification of surfaces have been carried out to make it hydrophobic/hydrophilic [13,14]. In this context, designing materials with alternate slip and no-slip are useful so that drag reduction occurs and hence there is a possibility of reducing material damage. Kamrin et al. [15] have considered such phenomena and studied the importance of effective slip. Alternating no-slip and infinite slip was used for flow in a cylinder by Lauga and Stone [16]. Hendy et al. [17] have introduced a novel slip boundary condition by varying the slip length along the principal direction of a flow. Further, they have observed a good agreement between theory and results based on molecular dynamics simulations. The concept of patterned slip has been used recently by Zhao and Yang [18], while studying electro-osmotic flows in micro-channels. There is now a huge interest in methods for manipulating the behavior of fluids for probable use in small, integrated devices for performing various tasks: (a) manipulations at the cellular length scale (and below) and the ability to detect small quantities and manipulate very small volumes (typically less than 1 μl) [19–22], and (b) use in microsystems to perform fundamental studies of physical [23], chemical [24], and biological processes [25–27]. These types of investigations of fluid flows have reinvigorated interest in a classical area of fluid dynamics, low Reynolds number flows.

In this work, we use Stokes equation, which provides a very good description of fluid flows at low Reynolds numbers, to analyze fluid flows through micro-channels. The direct biharmonic boundary integral equation, discussed by Kelmanson [28], is used to investigate the steady, pressure-driven, two-dimensional flow of a Newtonian fluid through slip-patterned channels in the low Reynolds number limit. The slip condition at the boundary is imposed by using the Navier's slip boundary condition.

2. Mathematical formulation

The flow of a two-dimensional viscous incompressible fluid at low-Reynolds number is governed by the steady Stokes and continuity equation which in the respective non-dimensional forms are,

$$\nabla P = \nabla^2 \mathbf{U}, \quad \nabla . \mathbf{U} = 0, \tag{1}$$

where $\mathbf{U}$ and P denotes velocity and pressure respectively. The non-dimesionalization is done as follows:

$$\mathbf{U} = \frac{\mathbf{u}}{\widetilde{U}}, \quad \mathbf{X} = \frac{\mathbf{x}}{L}, \quad P = \frac{pL}{\mu \widetilde{U}}.$$

The mass conservation equation $\nabla . \mathbf{U} = 0$ allows one to introduce stream function given by $U = \frac{\partial \psi}{\partial Y}$, $V = -\frac{\partial \psi}{\partial X}$, where (U,V) denote velocity components in (X,Y) cartesian coordinate system. Eliminating pressure using Stokes stream-function, Stokes equation reduces to a biharmonic equation, which is given by,

$$\nabla^4 \psi = 0. \tag{2}$$

It may be noted that there are variant approaches to solve Stokes equation using boundary element method, namely primitive variables (Stokesless formulation), stream-function vorticity formulation, etc. In general, the Stokesless formulation is expected to give very accurate results, that involve Green's function corresponding to velocity and stress tensor. On the other hand, stream-function vorticity variables formulation involves Green's function corresponding to Laplacian and biharmonic operators. However, this method suffers some disadvantages which are (i) difficult to extend in 3-D, (ii) unable to compute pressure filed explicitly, (iii) not suitable for the problems which involve the corner singularity. In order to remove the effect of the corner singularity either (i) large number of boundary elements can be used for discretization, or (ii) incorporating the analytic nature of the singularity and modifying the BBIE method. In spite of these disadvantages, the stream-function vorticity variable formulation was used widely.

We use the direct biharmonic boundary integral equation methods (BBIE) to solve Eq. (3). In the past, several researchers have discussed BBIE methods [28–36] in significant detail. Therefore, in this study we present the aforementioned method concisely. In order to solve Eq. (2) using the boundary element method, the biharmonic equation is rewritten in terms of stream function-vorticity variables,

$$\nabla^2 \psi = -\omega, \quad \nabla^2 \omega = 0. \tag{3}$$

Let G^L and G^B be the fundamental solutions of Laplace's and biharmonic equations respectively, which satisfies $\nabla^2 G^L = \delta(|q-p|)$, and $\nabla^4 G^B = \delta(|q-p|)$, where δ is the dirac delta function and $G^L = log|p-q|$, and $G^B = |p-q|^2(log|p-q|-1)$ [37]. Let us denote a general field point by $p(X, Y)$ and an integration point on the boundary by $q(X_0, Y_0)$, so that $p \in \Omega \cup \partial\Omega$ and $q \in \partial\Omega$.

Applying the Green's second identity to Eq. (3), we obtain the following coupled system of integral equation at a general field point p:

$$\lambda(p)\psi(p) = \int_{\partial\Omega} \left[\psi(q) \frac{\partial G^L(p, q)}{\partial n_q} - G^L(p, q) \frac{\partial \psi(q)}{\partial n_q} \right] ds(q) - \frac{1}{4} \int_{\partial\Omega} \left[\omega(q) \frac{\partial G^B(p, q)}{\partial n_q} - G^B(p, q) \frac{\partial \omega(q)}{\partial n_q} \right] ds(q), \tag{4}$$

$$\lambda(p)\omega(p) = \int_{\partial\Omega} \left[\omega(q) \frac{\partial G^L(p, q)}{\partial n_q} - G^L(p, q) \frac{\partial \omega(q)}{\partial n_q} \right] ds(q), \tag{5}$$

where $\lambda(p)$ is defined by,

$$\lambda(p) = \begin{cases} 2\pi, & \text{if } p \in \Omega, \\ \pi, & \text{if } p \in \partial\Omega, \\ 0, & \text{if } p \notin \Omega \cup \partial\Omega. \end{cases} \tag{6}$$

The boundary $\partial\Omega$ is discretized into N constant elements $\partial\Omega_j$ containing the mid-element boundary nodes q_j $(j = 1, 2, \ldots N)$. Over each element, we approximate the values of ψ, $\frac{\partial \psi}{\partial n}$, ω, and $\frac{\partial \omega}{\partial n}$ by piecewise constant functions ψ_j, $\frac{\partial \psi_j}{\partial n}$, ω_j, and $\frac{\partial \omega_j}{\partial n}$, for $j = 1, 2, \ldots, N$.

Applying the discretized form of Eqs. (4) and (5) at the mid-point $p \equiv q_i \in \partial\Omega_i$ $(i = 1, 2, \ldots, N)$ of each element gives a coupled system of vector equations,

$$A_{ij}\psi_j + B_{ij}\frac{\partial \psi_j}{\partial n_q} + C_{ij}\omega_j + D_{ij}\frac{\partial \omega_j}{\partial n_q} = 0, \tag{7}$$

$$A_{ij}\omega_j + B_{ij}\frac{\partial \omega_j}{\partial n_q} = 0, \tag{8}$$

where the coefficient matrices A, B, C, and D are given by,

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