Engineering Analysis with Boundary Elements 73 (2016) 191–199

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Impedance-to-scattering matrix method for large silencer analysis using direct collocation

P. Wang, T.W. Wu*

Department of Mechanical Engineering, University of Kentucky, Lexington, KY 40506-0503, USA

ARTICLE INFO

Keywords:
Impedance matrix
Scattering matrix
Large silencers
Boundary element method

ABSTRACT

Large silencers used in industry usually have a very large cross section at the inlet and outlet. Higher-order modes will populate the inlet and outlet even at very low frequencies. A three-dimensional analysis tool, such as the finite element method or the boundary element method, must incorporate certain forms of modal expansion in order to consider the higher-order modes in the transmission loss computation. In this paper, the impedance matrix obtained from the sub-structured boundary element method is converted into the scattering matrix that relates the higher-order modes. Since there are always more boundary elements at the inlet and outlet than the total number of propagating modes, a least-squares procedure is used to convert the element-based impedance matrix into the modal expansion-based scattering matrix. The transmission loss of the silencer can then be computed from the scattering matrix if a certain form of the incident wave is assumed. Furthermore, a slightly rearranged form of the scattering matrix may also be used to combine subsystems in series connection.

1. Introduction

Silencers used in the power generation industry usually have very large dimensions. Even a single unit isolated from an array of bar silencers or tuned-dissipative silencers may still have a large cross section at the inlet and the outlet. The plane-wave cutoff frequency of the inlet and outlet ducts of such silencers can be less than a few hundred Hz, and the frequency range of interest normally goes up to 8000 Hz or above. Due to the low cutoff, the conventional four-pole transfer matrix is not valid, and more importantly the anechoic termination can no longer be represented by the characteristic impedance boundary condition. Although the silencer itself is often modeled by a three-dimensional analysis tool such as the boundary element method (BEM) or finite element method (FEM), a direct computation of the transmission loss (TL) from the BEM or FEM model can be challenging without incorporating certain forms of modal expansion. Kirby and his co-workers [1–5] used a hybrid FEM to study the acoustical performance of large dissipative silencers. To apply the hybrid technique, the 2D FEM is first employed to extract the eigenvalues and the associated eigenvectors of an axially uniform cross section. These 2D transversal modes are then used in the modal expansion along the axial direction if the cross section remains the same. To determine the unknown amplitudes in the modal expansion, either a point collocation method or a mode matching scheme is adopted to enforce the continuity of sound pressure and particle velocity at both ends where the uniform section meets the flanges or any irregular junctions. Higher-order modes, including some evanescent modes, are considered in the modal expansion because the evanescent modes are still important at the flanges or irregular junctions. Since the FEM is mainly used on a 2D cross section to extract the modes, the hybrid FEM is a very efficient numerical technique for silencers with a very long axially uniform section. Due to the large cross sections at the inlet and outlet for such applications, Mechel [6] suggested three different source models, constant modal amplitude, equal modal power, and equal modal energy density. The equal modal energy density source model is believed to be most realistic.

On the BEM side, Zhou et al. [7] recently proposed a reciprocal identity method in conjunction with the BEM impedance matrix to extract the higher-order modes at the inlet and outlet of an axisymmetric silencer. Each reciprocal identity couples the analytical modal expansion in the inlet and outlet ducts to a BEM solution with a random boundary condition set. The modal expansion assumes a certain form of the incident wave in the inlet duct, and an anechoic termination at the outlet. The unknown modal amplitudes are the reflected waves in the inlet duct and the transmitted waves in the outlet duct. Depending on how many modes can propagate to the inlet and outlet at a given frequency, a minimum number of BEM solutions are needed for the reciprocal identity coupling. The BEM impedance matrix can naturally provide more than enough such solutions since

* Corresponding author.
E-mail address: timwu@engr.uky.edu (T.W. Wu).

http://dx.doi.org/10.1016/j.enganabound.2016.09.014
Received 23 May 2016; Received in revised form 18 September 2016; Accepted 28 September 2016

each column of the impedance matrix represents a BEM solution corresponding to a unique boundary condition set. A least-squares procedure is then used to solve for the unknown modal amplitudes.

In this paper, a so-called "impedance-to-scattering matrix method" is proposed to extract the modes at the inlet and outlet from the BEM impedance matrix. Compared to the reciprocal identity method, the proposed method is a collocation approach that directly converts the BEM impedance matrix into the scattering matrix for TL computation. The BEM impedance matrix relates the sound pressures at the inlet and outlet to the corresponding particle velocities, while the scattering matrix relates the modes at the inlet and outlet [8]. In a circular or rectangular duct, each sound pressure and particle velocity can be expanded in terms of the analytical duct modes at the centroid of each constant boundary element. These point-wise expansions are then related by the BEM impedance matrix, and the scattering matrix can be obtained after a few matrix operations. Normally there are more boundary elements than the total number of modes at the inlet and outlet, and a least-squares procedure is used to condense the element-based impedance matrix to the mode-based scattering matrix. The TL computation will follow if a certain form of the incident wave is assumed and the outlet is non-reflective.

One bonus effect of producing the scattering matrix as a byproduct in addition to TL is that it will be easier to change to a more complex incident wave form and/or a more realistic termination condition at the outlet, should the need arise in the future. The scattering matrix may also be used to combine subsystems in series connection. The scattering matrix can be slightly rearranged into a so-called "transfer scattering matrix" that puts the modes at the inlet on one side and the modes at the outlet on the other. This is very similar to the way that the traditional four-pole transfer matrix is set up, except that the higher-order modes are now included in the scattering matrix.

The BEM impedance matrix used in the proposed method is based on the direct mixed-body BEM with a substructuring technique [9]. The direct mixed-body BEM can handle complex internal components [10,11] as well as multiple bulk-reacting materials [12,13] in one single BEM domain without resorting to the tedious multi-domain BEM. It relies heavily on the use of a hypersingular integral equation to accompany the Helmholtz integral equation. A review of hypersingular integrals can be found in Chen and Hong [14]. It should also be noted that the Helmholtz integral equation used in this study does not include any mean flow. Nonetheless, the flow effect can still be partially considered if an accurate perforate transfer impedance formula is used [11] and incorporated in the model as the boundary condition on perforated tubes. With the substructuring technique, a very large silencer can be divided into several smaller substructures at any cross sections along the axial direction of the silencer. Continuity of sound pressures and particle velocities at junctions is automatically enforced when the BEM impedance matrices are merged by a synthesis procedure [9]. After all the impedance matrices of the substructures are merged, the resulting impedance matrix of the silencer is used as the starting point of the proposed method. Therefore, like the reciprocal identity method, the proposed impedance-to-scattering matrix method can be regarded as a post-processing filter applied to the existing BEM impedance matrix in order to extract the higher-order modes at the inlet and outlet. The solver is the same sub-structured BEM solver as in Ref. [9]. In contrast, modal expansions have been fused into the BEM in the past for other applications, such as underwater waveguides [15,16] and ultrasound defect detection [17,18], and those hybrid techniques usually require coupling the modal expansions in the far field to the BEM in the near field. On the other hand, the proposed method does not require any modifications to the BEM itself as long as the BEM can produce an accurate impedance matrix. In fact, the impedance matrix is not "BEM-exclusive". In theory, the 3D FEM can also produce the impedance matrix if needed.

Several commonly used inlet/outlet configurations are considered in this paper. These include axisymmetric, non-axisymmetric circular, and rectangular inlet/outlet shapes. These cover most of inlet/outlet shapes used in industry. Any irregular inlet/outlet shapes will require an additional numerical procedure to extract the 2D cross-sectional modes first, and the topic will be discussed in a companion paper. Numerical results are validated by comparing to available analytical or approximate solutions for the axisymmetric and rectangular test cases. As for the non-axisymmetric circular configurations, an indirect validation procedure is adopted. A silencer with a small inlet and a small outlet is artificially divided into two subsystems at a junction that has a large cross section. The transfer scattering matrix of each subsystem can be obtained individually by using the proposed impedance-to-scattering matrix method. The TL obtained by multiplying two subsystem transfer scattering matrices together is then compared to the direct TL solution from the conventional BEM, which is very capable of producing an accurate result for mufflers with a small inlet and a small outlet. Due to the large cross section at the artificial junction between the two subsystems, the two scattering matrices with higher-order modes are indirectly verified.

2. BEM impedance matrix

The BEM impedance matrix **Z** is defined by

$$\begin{Bmatrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1q} \\ \cdots\cdots \\ p_{21} \\ p_{22} \\ \vdots \\ p_{2l} \end{Bmatrix} = \begin{bmatrix} Z_{1,1} & \cdots & Z_{1,q+l} \\ \vdots & \ddots & \vdots \\ Z_{q+l,1} & \cdots & Z_{q+l,q+l} \end{bmatrix} \begin{Bmatrix} v_{11} \\ v_{12} \\ \vdots \\ v_{1q} \\ \cdots\cdots \\ v_{21} \\ v_{22} \\ \vdots \\ v_{2l} \end{Bmatrix} \tag{1}$$

where p and v denote the sound pressure and particle velocity at the inlet and outlet. For p and v, the first subscript 1 represents the inlet and 2 the outlet; the second subscript represents the boundary element numbering (q constant elements at inlet and l constant elements at outlet). The impedance matrix can be obtained by "tuning on" v=1 on each element at the inlet and outlet successively, one at a time. Although there are a total of q+l velocity boundary condition sets, they all share the same BEM coefficient matrix. Therefore only one matrix inverse is needed at each frequency. Eq. (1) can re-written in a more compact vector form:

$$\begin{Bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \end{Bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{Bmatrix} \tag{2}$$

where 1 and 2 denote the inlet and the outlet, respectively, and the element numbering index is dropped.

Normally, it is not practical to solve a large silencer problem with one single piece of BEM mesh due to the size limit. Fig. 1 demonstrates how a tuned dissipative silencer, also known as the "pine-tree" silencer, can be divided into several small substructures so that each small substructure can be analyzed on a desktop computer. In particular, the impedance matrix of a small template taken from the middle section B can be repeatedly used. As shown in Fig. 1, only three small sections of the pine-tree silencer need to be modeled in the BEM: the beginning section (substructure A), the middle-section template (substructure B), and the tail section (substructure C). The majority part of the middle section does not need to be meshed at all because the impedance matrix of the template can be repeatedly used. An impedance matrix synthesis procedure [9] is then performed to combine all the substructure impedance matrices into a resultant impedance matrix for the whole silencer. The resultant impedance matrix, in the form of Eq. (2), can then be used as the starting point in the impedance-to-scattering matrix method.