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Finite element analysis enhanced with subdivision surface boundary representations

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ABSTRACT

In this work we develop a design-through-analysis methodology by extending the concept of the NURBS-enhanced finite element method (NEFEM) to volumes bounded by Catmull-Clark subdivision surfaces. The representation of the boundary as a single watertight manifold facilitates the generation of an external curved triangular mesh, which is subsequently used to generate the interior volumetric mesh. Following the NEFEM framework, the basis functions are defined in the physical space and the numerical integration is realized with a special mapping which takes into account the exact definition of the boundary. Furthermore, an appropriate quadrature strategy is proposed to deal with the integration of elements adjacent to extraordinary vertices (EVs). Both theoretical and practical aspects of the implementation are discussed and are supported with numerical examples.

1. Introduction

From a historical perspective, the development of Computer Aided Design (CAD) and numerical analysis, also referred to as Computer Aided Engineering (CAE), have followed different paths. The standard used to describe objects in CAD are boundary representations (B-reps). These B-reps are often given by parametric surfaces, commonly based on Non-Uniform Rational B-Splines (NURBS) [1]. On the other hand, in numerical analysis, the reference technique is the Finite Element Method (FEM), where the computational domain is discretized with a mesh of elements. Within each element, polynomial shape functions are constructed for the numerical approximation.

The use of different representations, employed by CAD and FEM, requires the intermediate step of mesh generation which, in many applications, can drastically slow down the entire analysis process. In addition, some geometrical features from the original CAD model may be lost in the FEM representation. As any design-through-analysis process would clearly benefit from a better integration of these two fields, uniting CAD and FEM has become an active topic of research. Major contributions towards this goal are represented by Isogeometric Analysis (IgA) [2,3] and the NURBS-Enhanced Finite Element Method (NEFEM) [4–7].

The key idea of IgA is to use the same basis functions for both the geometric representation and the numerical analysis. However, limitations arise when the framework is extended to analysis on volumes — whereas a boundary representation is sufficient for CAD purposes, a trivariate parametrization is required for the analysis. This parametrization is not straightforward to construct for complexly shaped objects because of the tensor product nature of NURBS discretizations. In addition, many CAD models are a collection of trimmed NURBS patches that, in general, cannot be stitched together without some small gaps or overlaps occurring at their interfaces.

In order to overcome the limitations imposed by the tensor product structure, other parametric boundary representations have been considered. One of these methods is represented by subdivision surfaces [8,9], allowing designers to model objects of arbitrary topology as a single watertight surface. Many subdivision schemes produce surfaces that can be numerically evaluated at arbitrary parameter values [10]. Subdivision surfaces were already used for numerical analysis before IgA was introduced [11,12]. More recent publications consider analysis on sub-

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division solids [13] and the local refinement of subdivision splines [14].

Another option is to use T-splines, which can be seen as a generalization of NURBS. The original T-splines [15] were developed to merge two NURBS surfaces defined using different knot vectors (resulting in T-sections in the mesh, hence the name T-splines). Later, they were combined with subdivision surfaces into T-NURCCS [16] using the NURSS framework [17]. More recent versions of T-splines have replaced the subdivision approach around irregular points in the control net (referred to as *extraordinary vertices* in subdivision terminology or *star points* in T-spline terminology) by G^1 biquartic patches [18]. An interesting property of T-splines is that they inherently allow local refinement of the control net. IgA based on T-splines [19] has developed considerably over the last few years and achieved several notable results, including the trivariate parametrization of genus-zero objects [20]. The subset of T-splines used in IgA are referred to as *analysis-suitable* T-splines [21].

In parallel to the development of IgA, NEFEM is another methodology that aims at reducing the gap between CAD and CAE. In the NEFEM approach, the shape functions are defined in the physical space and the exact boundary representation is taken into account in the numerical integration, due to a special volumetric mapping that includes the NURBS parametrization in its formulation [4]. In contrast to IgA, where the trivariate parametrization of complex shapes is one of the main issues, the mesh generation procedure used in the NEFEM framework follows those of standard FEM, paying special attention to capturing small geometric features [5]. The only condition that is assumed in the NEFEM literature is watertightness (G^0 continuity) at the interface of the different surface patches [7]. Unfortunately, as mentioned above, complicated CAD models often present some small inconsistencies at such interfaces, which results in complications with regard to FEM mesh generation. Although some strategies, often referred to as cosmetics, are available to improve the original CAD model and facilitate mesh generation, these processes are rarely automatic and require, in most cases, the supervision of the analyst [22,23].

Motivated by these issues, and taking into account the potential showed by subdivision surfaces in IgA [24–29], we extend the concept of NEFEM to subdivision surfaces. The main advantage of the approach is that the mesh can be automatically generated given a subdivision surface modeled in a CAD environment. Like in the case of NEFEM, the exact geometry is considered in the analysis and the original CAD representation is preserved if p-refinement is applied to the FEM approximation. This is not the case in standard isoparametric finite elements, where the error introduced in the discretization remains also after p-refinement, unless a new mesh is created. At the same time, if h-refinement is required, the generation of a new mesh is facilitated by the watertightness of the surface. In this way, even if a mesh is still used, the conversion between the CAD and the FEM models is no longer a bottleneck.

Although many subdivision algorithms are available, in this work we limit our attention to the Catmull-Clark scheme [30], which is the most widely used scheme in both computer graphics and IgA. The outline of the paper is as follows. An overview of subdivision surfaces, with emphasis on the Catmull-Clark scheme, is presented in Section 2. Section 3 describes the procedure employed for the mesh generation, while the NEFEM formulation and its extension to subdivisions surfaces are considered in Section 4. It includes a strategy to handle numerical integration around extraordinary vertices in the surface mesh. Section 5 discusses various numerical examples. Finally, some concluding remarks are given in Section 6.

2. Subdivision surfaces

Subdivision surfaces are a powerful tool for designing objects of arbitrary topology [31]. While they represent the leading technology for modeling free-form shapes in animated movies [32] and are a common modeling primitive for 3D games, the use of subdivision surfaces in CAE is still rather limited. The main reason for this is that most CAD models used for CAE are based on NURBS. This standard allows for the design of models including exact conical sections, something which can only be *approximated* by traditional subdivision schemes. However, with the advent of NURBS-compatible subdivision surfaces [33] and the possibility to convert trimmed NURBS surfaces to subdivision surfaces [34], the gap between these two standards is closing rapidly. Integrating subdivision surfaces in a CAD/CAE environment has become a topic of active research [35].

Similar developments can be seen in commercial CAD/CAE software packages. Notable examples include the introduction of the *Realize Shape* environment in NX 9, the *Mesh Modeler* in AutoCAD 2010, the *Imagine & Shape* module in CATIA 5 and the *Freestyle* extension in PTC Creo, which are all based on subdivision surfaces.

The use of subdivision surfaces in CAE was first proposed in Ref. [11]. Other work in a similar framework, by then referred to as IgA, includes [24–28]. Solving partial differential equations (PDEs) on subdivision *volumes*, as opposed to solving PDEs on subdivision *surfaces*, is a direction of research only just initiated [13,36]. It is here that we see the added value of extending the concepts of NEFEM to subdivision surfaces, as it facilitates the numerical analysis on volumes bounded by a subdivision surface.

In this paper we focus on the Catmull-Clark subdivision scheme [30], which is described in the remainder of this section. Combining the approach discussed in this paper with other subdivision schemes based on box-splines [37] is straightforward.

2.1. Mesh refinement and smoothing

From a designer's point of view, a Catmull-Clark subdivision surface can be interpreted as the result of indefinitely refining and smoothing an initial mesh $\mathcal{M}_0$. Such a mesh, also referred to as the *control net*, consists of vertices $\mathcal{V}$, edges $\mathcal{E}$ and quadrilateral faces $\mathcal{F}$.

The refinement and smoothing rules are represented by a set of affine combinations referred to as *stencils*, indicating how vertices in a mesh $\mathcal{M}_i$ should be weighted in order to obtain the mesh $\mathcal{M}_{i+1}$. Repeatedly applying the set of stencils to an initial mesh $\mathcal{M}_0$ yields a sequence of meshes $\mathcal{M}_0, \mathcal{M}_1, \dots, \mathcal{M}_i, \dots, \mathcal{M}_\infty$. Careful selection of the stencils results in the convergence of this sequence to a smooth surface $\mathcal{M}_\infty$ which is referred to as the *limit surface*. Fig. 1 illustrates a couple of steps of the Catmull-Clark scheme. Note that for graphical applications it is often sufficient to subdivide only a few times — the actual limit surface might not be relevant for such purposes.

2.2. Masks and stencils

From a mathematical point of view, the Catmull-Clark scheme is a generalization of midpoint knot-refinement for uniform bicubic B-spline surfaces. In the univariate setting, uniform B-spline basis functions can be defined using a knot-vector Ξ. This knot-vector is chosen to be a set of consecutive integers, in other words, $\Xi = [j, j+1, \dots, k] \subset \mathbb{Z}$. Midpoint knot-refinement consists in inserting the relevant half-integers $\mathbb{Z} + \frac{1}{2}$ into the knot-vector. Repeating this process i times results in a knot-vector $\Xi \subset \mathbb{Z}/2^i$.

Uniform B-splines defined on $\mathbb{Z}/2^i$ can be composed of scaled, shifted versions of themselves, each version scaled by a coefficient [38]. In this context, *scaled* refers to the smaller versions of the uniform B-splines defined on $\mathbb{Z}/2^{i+1}$. The concept is illustrated in Fig. 2 for the uniform cubic B-spline $N_4(t)$, where the subscript indicates the *order* (defined as *degree*+1) of the B-spline.

When written out, we obtain the expression

$$N_4(t) = \frac{\boxed{1}}{8} N_4(2t) + \frac{\boxed{4}}{8} N_4(2t-1) + \frac{\boxed{6}}{8} N_4(2t-2) + \frac{\boxed{4}}{8} N_4(2t-3) + \frac{\boxed{1}}{8} N_4(2t-4). \quad (1)$$