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2D and 3D Abaqus implementation of a robust staggered phase-field solution for modeling brittle fracture

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ABSTRACT

In order to model brittle fracture, we have implemented a two and three dimensional phase-field method in the commercial finite element code Abaqus/Standard. The method is based on the rate-independent variational principle of diffuse fracture. The phase-field is a scalar variable between 0 and 1 which connects broken and unbroken regions. If its value reaches one the material is fully broken, thus both its stiffness and stress are reduced to zero. The elastic displacement and the fracture problem are decoupled and solved separately as a staggered solution.

The approach does not need predefined cracks and it can simulate curvilinear fracture paths, branching and even crack coalescence. Several examples are provided to explain the advantages and disadvantages of the method. The provided source codes and the tutorials make it easy for practicing engineers and scientists to model diffuse crack propagation in a familiar computational environment.

1. Introduction

Fracture is one of the main failure modes for engineering materials. However, most of the time design codes apply large safety factors to avoid its manifestation. Additionally, to the devastating consequence of a brittle failure, their evolution is difficult to study in practice. Therefore, predicting the initiation and the propagation path of a fracture is of great importance for practicing engineers and scientists.

The original theory to understand brittle crack evolution was introduced first by Griffith [1], then a new metric, called the stress intensity factor, was proposed by Irwin [2] to account for the microscopic plasticity near the crack tip, even for macroscopically brittle materials [3,4]. They considered crack propagation as a stability problem: if the energy release rate reaches a critical value, the crack is able to open. The original theory describes crack propagation adequately, but it is insufficient to account for initiation, curvilinear crack paths, benching or coalescence.

Nowadays several methods are available to model crack propagation in solids. These methods can be categorized into two major groups depending on how they account for the supposed discontinuity: discrete or diffuse. Using discrete methods, such as node splitting [5], cohesive surfaces [6], hybrid discrete and finite element methods [7], the crack can only propagate between elements, therefore its path is strongly mesh dependent. This problem was overcome by the group of Belytschko [8,9] using a local enrichment in the shape functions of a finite elements (XFEM), as well as by Grüses and Miehe [10] with a configurational-force-driven sharp fracture front.

The second group of fracture modeling assumes that the discontinuity in the material is not sharp, but can be interpreted as a smeared damage. This theory led to the development of the phase-field model [11,12]. This way, the weakness of the original approach of Griffith can be overcome by a variational approach based on energy minimization, as proposed by several authors [13–17]. These approaches introduce a regularized sharp crack taken into account by an auxiliary scalar damage variable. This variable is considered as a phase-field establishing the connection between intact and broken materials.

Over almost a decade this method has gained significant visibility due to its flexible implementation. Besides the work of Msekh et al. [18], mostly in-house softwares were developed to model fracture with phase-fields. Unfortunately, the aforementioned paper neglects to reproduce the results of most of the previous implementations [12], and its source code is not available.

In this paper we give a fully functional implementation as an Abaqus/Standard UEL [19] of the phase-field model [20] to study the quasi-static evolution of brittle fracture in elastic solids. Additionally, as a Supplementary material the source code for the UEL and several examples are provided. Our purpose is to make the diffuse crack propagation scheme widely available not only for numerical scientists,

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but for practical applications and design engineers as well.

Furthermore, the provided source codes can be easily developed to account for dynamical effects [21], large deformations [22], path-following [23] or multi-physics problems [24,25]. One of the major advantage of present implementation, is that no additional updates and softwares are necessary, but only the widely available Abaqus/Standard [19] package and a FORTRAN compiler. It can fit into any existing platform and can be parallelized easily.

The quasi-static simulation of brittle fracture phase-field problem is solved using a staggered algorithm [20]. This approach decouples the elastic and the fracture problem. The strategy has proven to been computationally efficient and extremely robust. However, to reach an accurate solution the step size should be chosen carefully.

Our results compare favorably with the originally developed algorithm [20], as well as with other methods. We provide several examples both with the Abaqus input and FORTRAN files for better understanding and further development. The implementation contains 2D plane strain and 3D cases as well.

The paper is structured as follows. In Section 2 the difference between sharp and diffuse (phase-field) crack is explained. Then the coupling between the elastic solution and the phase-field problem is unfolded. Finally the staggered solution and its finite element implementation are given. Section 3 gives numerous examples and benchmark tests to validate and understand the simulation process. We also highlight the effect of most of the numerical parameters, such as the time step, length scale parameter or even mesh density. Finally in Appendix B a detailed description is given to guide the users in the development of their own models.

2. Methods

2.1. Phase field approximation of diffuse crack topology

To introduce the concept of a diffuse crack topology, let us consider an infinite one directional bar aligned along the x axis with a cross section Γ (see Fig. 1a). Let us assume a fully opened crack at x=0. If function $d(x)$ describes the damage, a sharp crack shown in Fig. 1b is a Dirac delta function. Its value is zero everywhere except at x=0, where

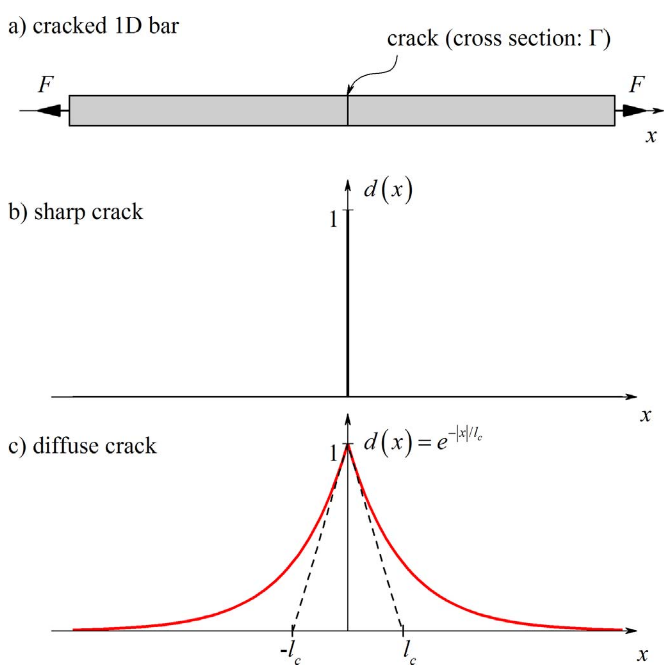

Fig. 1. (a) 1D bar with a crack at the middle with the cross section Γ. (b) Phase-field for sharp crack at x=0. (c) Diffuse crack at x=0 modeled with function (1) and length scale parameter l_c.

$d(0) = 1$. Variable $d(x)$ is the crack phase-field function. If its value is zero, the material in unbroken, if its value reaches 1, it is fully broken.

Following the idea that the crack itself is not a discrete phenomenon, but initiates with micro-cracks and nano-voids, we introduce an exponential function to approximate the non-smooth crack topology:

$$d(x) = e^{-|x|/l_c}, \tag{1}$$

where l_c is the length scale parameter and $d(x)$ represents the regularized or diffuse crack topology. Basically, with this idea the sharp crack is diffused as shown in Fig. 1c. By $l_c \to 0$ the sharp case is recovered. Function (1) has the property $d(0) = 1$ and at the limits $d(\pm\infty) = 0$.

It is the solution for the homogeneous differential equation [12]:

$$d(x) - l_c^2 d''(x) = 0 \quad \text{in} \quad \Omega, \tag{2}$$

subject to the Dirichlet-type boundary condition shown above. The variational principle of strong form (2) can be written as:

$$d = \mathrm{Arg}\left\{\inf_{d\in W} I(d)\right\}, \tag{3}$$

where

$$I(d) = \frac{1}{2}\int_\Omega (d^2 + l_c^2 d'^2)dV, \tag{4}$$

and $W = \{d|d(0) = 0,\ d(\pm\infty) = 0\}$. Now observe that the integration over volume $dV = \Gamma dx$ gives $I(d = e^{-|x|/l_c}) = l_c\Gamma$. Thus, the fracture surface is related to the crack length parameter. As a consequence, we may introduce a fracture surface density with the help of the phase-field function by:

$$\Gamma(d) = \frac{1}{l_c}I(d) = \frac{1}{2l_c}\int_\Omega (d^2 + l_c^2 d'^2)dV = \int_\Omega \gamma(d, d')dV, \tag{5}$$

where $\gamma(d, d')$ is the crack surface density function in 1D. Similarly, in multiple dimensions it can be expressed as:

$$\gamma(d, \nabla d) = \frac{1}{2l_c}d^2 + \frac{l_c}{2}|\nabla d|^2. \tag{6}$$

It can be seen that the gradient of the phase-field plays a significant role in the description.

2.2. Strain energy degradation in the fracturing solid

To couple the fracture phase-field with the deformation problem, we can write the potential energy of a solid body as:

$$\Pi^{\mathrm{int}} = E(\mathbf{u}, d) + W(d), \tag{7}$$

where $E(\mathbf{u}, d)$ is the strain and $W(d)$ is the fracture energy. Let $\Omega \subset \mathbb{R}^\delta$, be the reference configuration of a material body with dimension $\delta \in [1-3]$, and $\partial\Omega \subset \mathbb{R}^{\delta-1}$ its surface. The crack and the displacement field is studied in the range of time $T \subset \mathbb{R}$. Consequently we can introduce the time dependent crack phase-field:

$$d: \begin{cases}\Omega \times T \to [0, 1] \\ (\mathbf{x}, t) \to d(\mathbf{x}, t).\end{cases} \tag{8}$$

and the displacement field:

$$\mathbf{u}: \begin{cases}\Omega \times T \to \mathbb{R}^\delta \\ (\mathbf{x}, t) \to \mathbf{u}(\mathbf{x}, t).\end{cases} \tag{9}$$

In Eq. (7), the internal potential can be written:

$$E(\mathbf{u}, d) = \int_\Omega \psi(\boldsymbol{\varepsilon}(\mathbf{u}), d)dV, \tag{10}$$

where $\psi(\boldsymbol{\varepsilon}, d)$ is the potential energy density:

$$\psi(\boldsymbol{\varepsilon}, d) = g(d)\cdot\psi_0(\boldsymbol{\varepsilon}). \tag{11}$$

$\psi_0(\boldsymbol{\varepsilon})$ is the elastic strain energy and $g(d)$ is a parabolic degradation