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Implementation of topological derivative in the moving morphable components approach

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A B S T R A C T

We propose a new topology optimization approach based on the moving morphable components (MMC) framework with an explicitly described a layout through a finite number of components. The position and shape values of each component were defined as design variables. In this study, a method was developed by utilizing topological derivative. Instead of performing a discrete sensitivity analysis based on finite element methods, a topological derivative was used to calculate the first derivative of an objective function with respect to the shape and position of the components. The obtained derivative was validated via discrete sensitivity analysis. The topological derivative formulation has been well developed in recent years for different structural and non-structural problems. Utilizing this powerful tool enabled the MMC approach to easily solve various types of topology optimization problems. Herein, the presented method is illustrated through several topology optimization problems such as stress-based and thermo-mechanical topology optimization.

1. Introduction

The objective of this work is to develop a moving morphable components (MMC) framework approach based on the topological derivative concept. The MMC approach utilized a finite number of movable and deformable components to define the layout of a structure. By moving or changing the shape of these components during the optimization process, some empty spaces were either created in the design domain or were filled with materials (Fig. 1). On the other hand, a topological derivative is defined as the effect of an infinitesimal change in topology with regard to the quantity of an objective function. This topological change could be the insertion of a small hole in the domain or the addition of a small amount of material to the structure layout. Therefore, the concept of a topological derivative could be utilized in the MMC approach if we were to calculate the topological derivative during changes in position or shape of the components.

1.1. Topology optimization methods

Topology optimization (TO) is one of the most popular methods used for structural optimization, having rapidly extended from academic research to industrial applications [1]. The TO method fundamentally optimizes the geometry over arbitrary domains. This method was introduced via the Homogenization approach [2], in which varying material properties in space are described by composite materials. Later, TO was developed via two popular strategies, the solid isotropic material with penalization (SIMP) method [3] and topology optimization based on the level-set method [4–6]. There are also new alternative approaches with regard to TO [7,8].

In the SIMP method, the values of pseudo-densities assigned to elements were found to minimize the objective function [9]. The objective function in the majority of structural optimization studies was compliance [10], but other practical cost functions such as stress, displacement, and natural frequency were considered by the SIMP method. The SIMP method was relatively easy to implement [11] and well-developed, being utilized to solve structural and non-structural multi-physics systems [12–16].

As the SIMP method uses the pseudo-density in each element of the finite element method, the obtained layout possessed jagged shapes at the boundaries. By contrast, in the level-set based method, the design domain was specified by a surface defined by a level-set function; therefore, smooth boundaries could be obtained [5,17,18]. Indeed, the interface between material phases was defined implicitly by iso-contours of the scalar level-set function at all times, so the domain was well defined and singularity problems did not arise [19]. There were also many formulations and implementations of the level-set based method [20].

Several common problems in engineering have been considered in the TO. For example, one important problem involved a consideration

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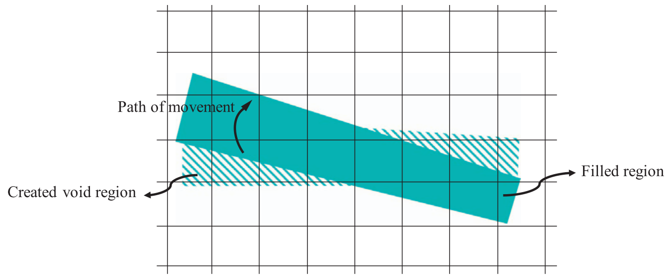

Fig. 1. Movement of a component in the design domain and background mesh.

of stress constraints in the design process [10]. SIMP and level-set based methods were well investigated with regard to solving stress constrained TO problems [21–23]. After the earliest work by Yang and Chen [24], several problems were addressed in the stress constrained TO. These issues could be categorized as singularity issues, highly nonlinear behaviors, and the local behavior of stress constraints [25]. Several methods such as epsilon-relaxation [26], qp-relaxation [27], relaxed stress indicators [28], Kreisselmeier–Steinhauser functions [29], and p-norms [27,30] were utilized to overcome the aforementioned problems.

Another common type of engineering problem considered in TO is thermo-mechanical problems. The earliest works in this field date back to 1995 [31]. The authors of [31] addressed the strong dependency of optimum design on temperature differentials. Xia and Wang [32] used the level-set based method to consider thermal effects with regard to structural optimization. In their study, the mean compliance was minimized and a geometric energy term was introduced to obtain a smooth boundary. Despite an easy implementation of the SIMP method in thermo-mechanical problems, zero density in elements required careful treatment such as the epsilon-method [26] due to singularity issues. Deaton et al. [33] demonstrated that typical compliance minimization in thermo-elastic problems may not generate favorable design, the reason being due to the design-dependency of the thermal load, which was subject to thermo-elastic effects during topology optimization. Moreover, compliant mechanism problems were solved by considering thermal effects in [34].

In both the SIMP and the level-set based method, an implicit definition of the boundary was obtained, which yielded some difficulties. For example, it is important to possess shape feature control during manufacturing [35,36], which is difficult in implicit ways due to special techniques that are necessary for length scale control. Moreover, the structural geometry needed in computer-aided design (CAD) is different from what is represented by implicit ways. This made it difficult to establish a link between CAD and the obtained layout during the optimization process [37]. Another drawback of implicitly defining the layout is the large number of required design variables, especially in 3D problems. By contrast to common methods in TO such as the SIMP and the level-set based methods, there is a new approach referred to as the moving morphable components (MMC) method, in which boundaries are explicitly defined by a polynomial function [8]. This method is described in the following sections and in Section 2.1.

1.2. MMC approach

A novel and practical approach in TO, referred to as the moving morphable components method, utilizes morphable components that can move and reshape to find the optimum layout of a structure under the given boundary conditions [8,38–41]. Indeed, this approach could be a bridge between size, shape, and topology optimization. The shapes of components were defined by an explicit boundary, which were functions of a finite number of variables [8]. These variables for all components in addition to the positions of the components were design variables during the optimization process. For example, one could use some bars with a constant thickness as components to define a 2D layout within the design domain (D). This design domain is presented as a dashed line in Fig. 2. In this example, the layout was explicitly defined by four bars and the number of design variables for each component was five: the x and y coordinates of the bar center and the length (L), thickness (t), and angle of the bar with a horizontal axis (θ). The total number of design variables for this example was $4 \times 5 = 20$. This requirement to lower the number of design variables defining a layout was another advantage of the MMC approach. Obviously, one could use components with different shapes and design variables, but the boundary of the layout was still explicitly defined. Additional parameters to define the shapes of components led to more flexible shapes for the components, but would increase computational costs.

1.3. Sensitivity analysis and topological derivative

Design sensitivity analysis is a crucial issue in the field of topology optimization. This analysis is used to compute the rate of change of a cost function, such as a change in strain energy or stress, with respect to changes in the design variables. Sensitivity analysis guided the optimization algorithms (i.e., SLP, MMA) to redistribute material within the design domain to determine the optimum layout. There are three approaches in the design sensitivity analysis: the approximation, discrete, and continuum approaches [42]. The approximation approach utilized finite difference methods to calculate design sensitivity. In the discrete approach, discrete FEM governing equations are used to obtain derivatives. In this approach, taking the derivative of the stiffness matrix is always necessary. This approach is widely used during topology optimization, particularly in the SIMP method. The continuum approach in design sensitivity analysis took the design derivative of the variational equation before it was discretized. One of the most powerful methods in this approach is the topological derivative.

The topological derivative concept was proposed in [43] and was later developed in several studies [44,45]. It possessed several applications with regard to shape and topology optimization, image processing, and mechanical modeling [46]. This concept expressed changes in

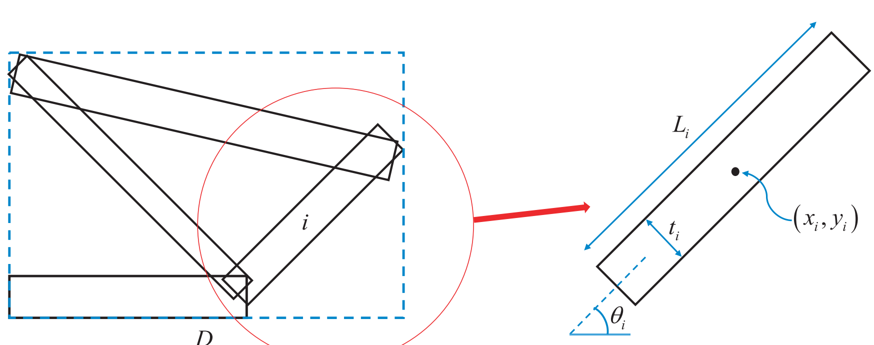

Fig. 2. Defining the layout of a structure with bars.