Finite Elements in Analysis and Design 133 (2017) 10–24

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

A new robust quadrilateral four-node variable kinematics plate element for composite structures

T.H.C. Le, M. D'Ottavio*, P. Vidal, O. Polit

UPL, Univ Paris Nanterre, LEME, 50 Rue de Sèvres, 92410 Ville d'Avray, France

ARTICLE INFO

Keywords:
Plate finite element
Variable kinematics model
Shear locking
Field compatibility approach
Mesh distortion
Composite structure

ABSTRACT

This paper presents a new four-node quadrilateral finite element for composite plates. A large number of displacement-based, variable kinematics plate models are formulated in the framework of Carrera's Unified Formulation, which encompass Equivalent Single Layer as well as Layer-Wise models with a displacement field that is defined by polynomials up to 4th order along the thickness direction z. The main novelty consists in the formulation of a field compatible approximation for the transverse shear strain field, referred to as QC4 interpolation, which eliminates the shear locking pathology by constraining only the $z-$ constant transverse shear strain terms. Extensive numerical studies are proposed that demonstrate the absence of spurious modes and of locking problems as well as the enhanced robustness with respect to distorted element shapes in comparison to classical isoparametric approaches. The new QC4 variable kinematics plate element displays excellent convergence rates under different boundary and loading conditions, and it yields accurate displacement and stresses for both, thick and thin composite plates.

1. Introduction

The increasing use of composite laminates and sandwich structures in engineering applications drives the need for appropriate analysis and design tools with dedicated computational models. Based on geometric considerations, composite panels are conveniently modeled as two-dimensional plate/shell structures. However, complicating effects - such as anisotropy, heterogeneity and transverse shear compliance - call for plate/shell models that go beyond the so-called *classical models*, i.e., those relying on (i) Kirchhoff-Love assumptions and neglecting transverse deformation (Classical Laminate Plate Theory, CLPT), (ii) or on Reissner-Mindlin assumptions and retaining a merely constant transverse shear deformation through the thickness (First-order Shear Deformation Theory, FSDT). Numerous review papers devoted to high-order plate/shell models witness the scientific progress in this specific topic [1–5].

A useful classification discerns Equivalent Single Layer (ESL) and Layer-Wise (LW) models [6]: In the former model class, the number of unknowns is independent of the number of layers constituting the composite plate, while in the latter one, the number of unknowns increases with the number of constituting layers. CLPT and FSDT evidently pertain to the ESL models. ESL models that enhance the kinematics for the transverse shear deformation while still discarding the transverse normal deformation, are referred to as High-order Shear Deformation Theories (HSDT). Contrary to FSDT, no numerical shear correction factors are required in HSDT thanks to an at least parabolic transverse shear distribution that may also exactly verify the stress boundary conditions at the laminate's top and bottom surfaces. Seminal examples are the third-order theory by Reddy [7] and the sinus-model by Touratier [8]. Conventional ESL models employ a single approximation for the displacement field through the entire laminate's thickness; the resulting transverse strain field is continuous across the stacked layers, which contradicts the equilibrium conditions between adjacent plies with different material properties [9]. Zig-Zag models are special ESL models with a piece-wise continuous displacement field that allows to fulfill the interlaminar continuity conditions of the transverse shear stresses, see, e.g., [10,11]. The review paper by Carrera [12] offers a comprehensive discussion about the various approaches to Zig-Zag models. In presence of highly compliant core layers, or for accurately resolving local stress gradients whose wavelength is comparable to the plate thickness, it is necessary to further refine the model upon including the transverse normal deformation, see, e.g., the early contribution by Lo et al [13] and the more recent papers [14,15]. Retaining the full three-dimensional constitutive law is particularly useful for coupled problems, such as those related to thermo-mechanics [16].

In order to reduce the computational effort, it has been proposed to limit the use of expensive high-order models to those local regions

* Corresponding author.
E-mail address: michele.d_ottavio@u-paris10.fr (M. D'Ottavio).

http://dx.doi.org/10.1016/j.finel.2017.05.002
Received 28 February 2017; Received in revised form 24 April 2017; Accepted 7 May 2017

hosting the stress gradients of interest, while employing classical low-order models for the remaining large portions of the structure. Several approaches have been followed for coupling heterogeneous kinematics and their respective Finite Element (FE) approximations, see, e.g., the transition element proposed by Feng and Hoa [17] within their hybrid-mixed composite plate element formulations, the overlapping mesh approach based on Arlequin method [18], or the direct interface coupling proposed by Wenzel et al [19] on the basis of an extended variational formulation.

A very flexible manner for implementing such substructuring methods is to resort to so-called variable kinematics models and corresponding finite elements, as first pioneered by Reddy [20] and more recently systematically developed by Carrera and co-workers thanks to a dedicated Unified Formulation [21,22]. By virtue of an extensive index notation, Carrera's Unified Formulation (CUF) allows to implement a large number of finite plate/shell elements within a unique software, whose inputs provided at runtime define the actual plate/shell model to be used in the analysis [23]. Within CUF, classical displacement-based plate elements are defined through the choice of (*i*) the description at laminate level (ESL or LW), (*ii*) whether a Zig-Zag function is to be used or not, and (*iii*) the order N of the polynomial expansion assumed for the displacement field. The original version of CUF as presented by [24] has been implemented as Abaqus User Element with a dedicated Python-based preprocessing tool [25]; this package has been successfully employed in, e.g., [26,27] and forms the basis of the developments proposed in this paper.

From the computational point of view, the development of robust Finite Elements (FE) is required to cope with the adopted two-dimensional plate/shell model, see, e.g., the discussion by MacNeal [28] about the FE technology employed for Kirchhoff-Love and Reissner-Mindlin shell models. A general, highly predictive plate FE should not rely on numerical tuning coefficients, should have only six rigid body modes, for spurious zero-energy modes could be particularly detrimental, e.g., in non-linear analyses, and should be free from numerical pathologies that could degrade the solution's accuracy in case of distorted elements or extreme thickness ratios. The most characteristic example for this latter issue is transverse shear locking, a spurious over-constraint that dramatically underestimates the bending deformation of a thin, shear-deformable plate element. Several techniques have been devised for correcting the transverse shear locking pathology affecting FSDT-based plate/shell elements, most of which can be stated from hybrid-mixed approaches [29]. The most widespread techniques are reduced integration methods, which, however, require a dedicated stabilization for preventing spurious zero-energy modes [30], or so-called B-bar methods [31], in which a specific constraint is used for the transverse shear strain field. Different approaches have been followed for constructing this modified strain field, such as Kirchhoff mode [32], Assumed Natural Strain (ANS) [33,34], Mixed Interpolation of Tensorial Components (MITC) [35], the field-consistency paradigm [36], Discrete Shear elements [37] or Discrete Shear Gap (DSG) [38].

Variable kinematics plate model suffer transverse shear locking also, and the first employed countermeasures consisted in a selectively reduced quadrature [39,24,40]. Rectangular four- and nine-nodes plate elements have been been subsequently implemented upon extending the MITC approach to CUF-based high-order kinematics [41]. A four-node quadrilateral plate element has been proposed by Kulikov and Plotnikova [42] by resorting to a hybrid-mixed ANS approach in conjunction with a variable kinematics approach formulated in terms of Sampling Surfaces (SaS). In these works, *all* transverse shear strain terms issued from the high-order kinematics are constrained according to the adopted MITC or ANS approach. However, since high-order shear deformation terms depend on the plate thickness and will vanish in thin-plate limit, the locking behavior is produced by the first-order Reissner-Mindlin kinematics only. As a matter of fact, the convergence rates of CUF elements do not depend on the polynomial order N defining the plate kinematics [40].

Refined two-dimensional structural models including thickness change require special care for preventing thickness or Poisson locking, i.e., an overly stiff bending response that is produced if the transverse normal strain is not allowed to vary along the thickness direction [43]. Two class of remedies have been proposed for correctly resolving the Poisson coupling along the thickness: to retain an at least linearly varying transverse normal strain, either within an Enhanced Assumed Strain approach pioneered by Büchter et al [44], see also [45], or directly in the model kinematics [16]; or to modify the constitutive law for the bending contribution by referring to a generalized plane stress condition, as proposed, e.g., in [46].

Based on the outlined background, the present paper proposes a four-node quadrilateral element for variable kinematics displacement-based CUF plate models. A special transverse shear locking correction is formulated by referring to the field consistency paradigm and applied only to the constant, thickness-independent part of the transverse shear strain. For this, the method first proposed by Polit et al [47] for FSDT, and subsequently extended to a refined kinematics [16], is here further extended to high-order plate models with arbitrary kinematics. The resulting plate FE is implemented as User Element in the commercial package Abaqus, along with dedicated Python plug-ins for generating the model within the graphical interface of Abaqus. Following the recommendations expressed by MacNeal and Harder [48], extensive numerical tests are reported for demonstrating the absence of spurious mechanisms and assessing the element's accuracy for extreme thickness ratios as well as distorted meshes. The paper is organized as follows: the CUF-based variable kinematics approach is recalled in Section 2 and the new QC4 FE approximation is presented in Section 3. The numerical results are discussed in Section 4, where a comprehensive investigation is proposed that concerns the rank of the stiffness matrix, the robustness of the element with respect to length-to-thickness ratio and mesh distortion, as well as the accuracy of the predicted displacements and stresses for homogeneous and composite plates. Finally, Section 5 summarizes the main conclusions and proposes an outlook towards further studies.

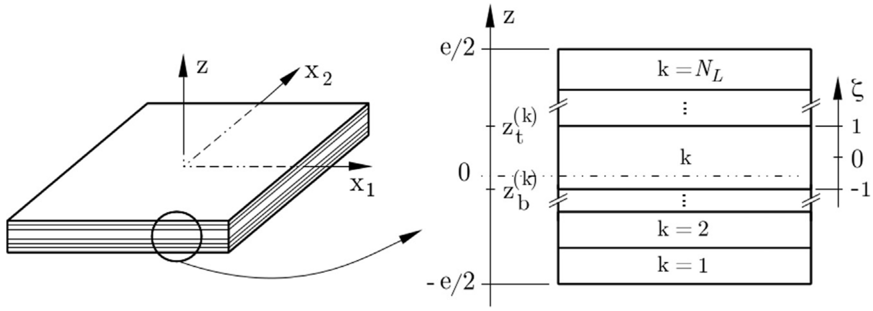

Fig. 1. Coordinates and notation used for the description of the composite plate.