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Finite deformation analysis of visco-hyperelastic materials via solid tetrahedral finite elements

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ABSTRACT

An alternative solid finite element formulation for large deformation analysis of viscoelastic materials is proposed. This new approach is based on positions and makes possible a robust implementation of an isoparametric solid tetrahedral that presents no locking when dealing with complex stress, strain and strain rate for general structural analysis. A consistent way to write internal variables that accounts for finite viscoelastic strains is proposed. In this alternative methodology the neo-Hookean hyperelastic law is taken into account together with the Zener viscoelastic model. The evolution law is described in terms of a rate equation involving the viscous right Cauchy-Green stretch tensor. The study is dedicated to homogeneous materials under isothermal and quasi-static conditions. The nonlinear solution procedure is performed via the Newton-Raphson iterative technique and the backward-Euler method.

Four illustrative examples involving large viscoelastic strains are analyzed: uniaxial tension, simple shear, buckling of a clamped column and partially loaded block. The present formulation can reproduce creep, stress relaxation and viscoelastic rate dependent stiffening at large strains, which are usually observed in polymeric materials. Even for very complex stress, strain and strain rates the mesh refinement of the proposed methodology leads to more accurate results, avoiding general locking problems. The effect of the viscosity parameter on the material response and the evolution of viscous stretches over time are also highlighted in the results.

1. Introduction

Viscoelastic materials have many practical applications in engineering, especially in polymeric structural components. These materials have the ability to creep, undergo stress relaxation and absorb energy. So, their main structural functions are impact absorption, noise reduction, damping system and vibration isolation. Several applications of viscoelastic polymeric materials can be cited: polymer foams used in seat cushions, helicopter acoustic blankets, automobile bumpers, shoe insoles, steel/polymer composite for damping systems, wrestling mats, foam padding inside helmets, among many others. The prediction of the mechanical behavior of these materials is, therefore, essential for design purposes.

The main difficulty arisen from the analysis of polymeric materials is that they usually present time-dependent highly nonlinear deformation in finite strain regime. In other words, polymers (or elastomers) are highly deformable and viscoelastic and usual problems regarding FEM implementation, as locking and mesh dependence, may be present. The finite elastic strains are usually treated in the context of hyperelasticity (see, for instance, [1]), and the time-dependence of the mechanical material response is described via viscoelastic models. Many viscoelastic formulations have been proposed in the scientific literature, including the small strain models and their corresponding extension to the finite strain regime. However, there is not a general finite viscoelasticity framework. The most common theoretical formulations, in its majority not implemented in finite elements, may be classified into three groups: Convolution Integral Model (CIM); Internal Variable Model with Linear Evolution Equation (IVM1); and Internal Variable Model with Nonlinear Evolution Equation (IVM2).

The first group (CIM) corresponds to the models in which the material viscoelastic response is described by convolution or hereditary integrals. According to [2], these models are related to the expansion theory of [3] and the extension of the Boltzmann superposition principle to finite strain regime has been performed by [4]. The strain

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(or the deformation gradient) is locally decomposed into a part that defines the deformation from time -∞ to τ, and a part that defines the deformation from τ to the actual time t. The stress is obtained by summing long-term low strain rate response (instantaneous elastic part) and the viscous overstress expressed in terms of time convolution integrals accounting for the material history [2].

The other two viscoelasticity frameworks (IVM1 and IVM2) are based on the original work of [5] and on the studies of [6] and [7]. The key idea of the internal variable models are the use of hidden (or internal) strain history variables, which cannot be observed or measured. The material response is defined by the total strain and the history variables, and the second law of thermodynamics (described by the Clausius-Planck inequality) is employed to set the internal dissipation and the stress-strain relation. The differences between these formulations are the history measure and the evolution equation employed. In the model IVM1, the evolution equation is set by a linear rate equation regarding the non-equilibrium stresses (or overstresses), which has a closed-form solution in convolution form and leads to a simple recursive update formula (see, for instance, the works of [8] and [9]). The strain decomposition into an elastic and a viscous parts is adopted only in the third model (IVM2), in which the evolution equation is defined by a rate equation involving the elastic left Cauchy-Green tensor (see, for example, [2] and [10–12]) or the viscous right Cauchy-Green tensor (see [13] and [14], for instance). A comparison of the models – regarding the large-strain viscoelastic response of polymers, thermodynamic consistence and numerical aspects – is provided in the study of [2], which concluded that the models differ for moderate strain rates and it is very difficult to determine which model must be preferred for specific applications. In the present study, the authors present a finite viscoelastic model, called visco-hyperelasticity, similar to the IVM2 formulation adapted to be used in a finite element environment. To develop our Finite Element Formulation we adapted the thermal-viscoelastic model for rubber-like materials proposed in [15], using a Helmholtz free energy function that neglects thermal effects.

Knowing the good performance of the fully integrated isoparametric solid tetrahedral finite elements of any-order proposed for finite elasticity by [16], we originally adapted the theoretical visco-hyperelastic model of [15] to develop a computer code for the analysis of highly deformable viscoelastic materials avoiding locking and mesh dependence for complex viscoelastic stress, strain and strain rate fields. There are several large-strain viscoelastic analyzes via finite elements in the scientific literature, but the use of solid elements are still limited and there is not the combination of the tetrahedral finite element and the proposed visco-hyperelastic model. For example, in the work of [17], 20-node solid and shell elements are employed to analyze traveling load problems in rolling, moving and rotating viscoelastic structures, and a contact algorithm is used to simulate a 15,000 degree-of-freedom model of a tire. This 20-node solid element is also used in [18] to analyze viscoelastic large deformation problems with a convolution model and to discuss the effects of material incompressibility on stress analysis. In the study of [8], an internal variable model together with assumed enhanced strain elements based on a three-field variational formulation is adopted, showing that the formulation can be used to reproduce creeping and relaxation phenomena for near-incompressible materials. A nonlinear viscoelastic response of reinforced elastomers is modeled in [13] using a 3D mixed finite element method with a nonlocal pressure field, and the resultant formulation can be used to simulate a solid propellant unit cell, as well as capture creep and relaxation phenomena. Second-order solid finite elements are employed in [12] to study viscoelastic and viscoplastic problems. A viscoelastic internal variable model based on logarithmic strain has been implemented by [14] into a user-defined subroutine in the nonlinear finite element software ABAQUS, using solid elements and showing good agreement with experimental data. As far as the authors' knowledge goes, there are no studies regarding large deformation analysis of viscoelastic materials via fully integrated solid tetrahedral finite elements, and no convergence analysis is performed for the existent formulations. By our experience the lack of convergence studies may be associated with locking and mesh dependence phenomena that are not present in the element proposed by [16] and [19], to be employed here.

The integration of the evolution equations, which is the update of the time-dependent variables, is an important issue concerning the viscoelastic models in numerical or finite element analyzes. The two most common time integration schemes are the backward-Euler and the exponential mapping schemes. The latter method, also used in elastoplasticity, guarantees viscous incompressibility, which is valid for many polymers, and allows larger time step sizes, but is very difficult to be implemented in a computer code for 3D formulations. Thus, the backward-Euler method has been adopted in the present study.

The purpose of the present study is to present an accurate and reliable numerical formulation, via solid tetrahedral finite elements, for the analysis of viscoelastic materials under finite deformations, finite strains, isothermal conditions and statically applied forces. In future works, it is intended to include the thermal dependence of the constitutive model due to its importance in polymer and some metal analyses.

The paper is organized as follows. The constitutive visco-hyperelastic model, written in a proper way to the numerical implementations, is described in the second section. The finite element approximation, including its positional aspect, is provided in Section 3. The numerical algorithm, as well as the time integration scheme, revealing the elegance of the proposed numerical strategy in aggregate new constitutive models, is given in the fourth section. The illustrative numerical examples are showed and discussed in Section 5, including simple and complex stress, strain and strain rate fields. Finally, the main conclusions are highlighted in the sixth section.

2. Visco-hyperelastic model

The finite viscoelasticity model described in the present section is based on the works of [10,13–15] and [20].

2.1. Deformation and strain

The deformation gradient, denoted by $\mathbf{F}$, is multiplicatively decomposed into an elastic and a viscous part, similarly to the Kröner-Lee decomposition employed in finite plasticity:

$$\mathbf{F} = \mathbf{F}_e\mathbf{F}_v \tag{1}$$

where the subscripts $()_e$ and $()_p$ denote, respectively, the elastic and the plastic parts. This decomposition in questionable in viscoelasticity according to [2], as the intermediate configuration defined by $\mathbf{F}_v$ can be considered an equilibrium configuration only if the time scale is much smaller than the relaxation time. This gradient tends to relax to the reference configuration as the time scale evolves, becoming elastic. Formulations in which decomposition (1) is adopted should then be used for quasi-static problems, as pointed out by [2].

Three more Lagrangian strain measures are decomposed in the present work based on (1):

$$\mathbf{C} = \mathbf{F}^T\mathbf{F} = \mathbf{F}_v^T\mathbf{C}_e\mathbf{F}_v \Rightarrow \mathbf{C}_e = \mathbf{F}_v^{-T}\mathbf{C}\mathbf{F}_v^{-1} \tag{2}$$