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# Topology optimization of pressure dependent elastoplastic energy absorbing structures with material damage constraints

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ABSTRACT

In this study, a density-based topology optimization framework for the design of energy absorbing structures with pressure-dependent yield behavior is presented. The plastic work is maximized while the accumulation of damage is managed through the use of macroscopic fracture constraints. Pressure-sensitive yield behavior is captured by the Drucker-Prager plasticity model and an adjoint method is presented to calculate the path-dependent sensitivities of the objective and constraint functions dependent on this model. Several numerical examples are used to demonstrate the effect of varying the pressure sensitivity of the yield function and the underlying physics is reflected in the final topologies. It is also demonstrated through numerical examples that the use of damage limiting constraints leads to optimal topologies with less damage localization for the same amount of plastic work.

## 1. Introduction

Energy absorbing structures are designed to dissipate energy under static or impact loading in a controlled manner. Structures of this type are needed in a wide variety of fields including crashworthiness designs, seismic designs and blast protection of buildings, and design of personal safety equipment such as hard hats, body armor and sports equipment, among others [1]. Despite the widespread need for energy absorbing structures, there is a lack of comprehensive guidelines for their design. To date, the design of energy absorbing structures is mostly ad-hoc, based on a combination of physical intuition and experience, while methods such as size and shape optimization have also been considered to improve designs [2–6]. In size and shape optimization, the overall form of the design is known a priori and only a small number of parameters are optimized to improve the design performance. In contrast, topology optimization methods that allow for the simultaneous optimization of size, shape and connectivity can provide a more holistic approach for designing these systems.

Topology optimization has been used in a number of different applications since the method was first proposed by Bendsøe and Kikuchi [7] in the late 1980s. The review studies in Refs [8–10] highlight the progress this field has made. The benefits of using topology optimization stem from the fact that it greatly expands the design space that can be explored in comparison to traditional size and shape optimization [11]. Thus, topology optimization systematically seeks out optimal forms that are not known a priori and offers a more rigorous design approach. For design applications where the goal is to absorb energy using irreversible plastic deformations in the underlying material, the optimal topology should maximize the plastic work dissipated under the applied loading conditions. However, the majority of the studies on topology optimization have focused on elastic material behavior with applications to maximum stiffness designs, designs for fundamental frequency and compliant mechanism designs, among others [8,11–23]. There are only a limited number of studies that consider plastic material behavior in topology optimization due to the challenges associated with the path-dependent nature of such inelastic materials. The existing literature involving the use of plastic materials in topology optimization includes the studies by Maute et al. [24], Schwarz et al. [25], Huang et al. [26], Kato et al. [27], Nakshatrala and Tortorelli [28], Wallin et al. [29] and the recent studies by the authors [30–32].

In all of the above studies, the plasticity models used in topology optimization consider yield potentials that are pressure insensitive. While these yield potentials are useful for the phenomenological description of polycrystalline metals, there are a number of practical materials whose plastic behavior cannot be captured by these models. Amorphous materials such as glassy polymers and metallic glasses show yield behavior that has a strong dependence on hydrostatic pressure, with different yield stresses under tension and compression. Moreover, amorphous glassy polymers have been shown experimen-

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tally to undergo significant plastic deformations [33]. One of the most commonly used pressure sensitive plasticity models is the Drucker-Prager model [34]. This model is commonly used to simulate inelastic behavior of granular materials such as soils, rocks and concrete, and has also been successfully used to model the plastic behavior of amorphous glassy polymers and glassy metals [35–37]. Drucker-Prager plasticity was used along with von Mises plasticity in the topology optimization study in Ref [38] where Voigt-Reuss bounds were used for material interpolation and structures were optimized for compliance minimization. In addition, Drucker-Prager plasticity was used by Bogomolny and Amir [39] for the conceptual design of reinforced concrete structures. In their study, a material interpolation scheme is presented that eliminates the influence of pressure in the yield function when the material is to be modeled as steel. In this way, structures are designed for maximum stiffness given a certain amount of steel material. While the above studies consider Drucker-Prager plasticity models, they both focus on stiffness designs rather than energy absorption.

When designing energy absorbing plastic structures, the goal is to maximize the plastic work. However, as a material deforms plastically, the accumulated plastic strains will lead to damage in the material. This damage is due to the presence and evolution of cracks and cavities at the microscopic level, which eventually lead to material failure and a complete loss of load carrying capacity [34]. Thus, accounting for such inelastic material damage is critical in topology optimization for energy absorption. In essence, the goal of topology optimization for energy absorption should be to ensure that plastic work is distributed in a way that the premature local material failures due to high-localized damage are obviated. Such premature failures will result in a loss of energy absorption capacity and thus hinder the overall design performance. The final failure will eventually occur by such localizations, but an optimal topology design should delay this to promote large energy absorption before failure occurs. Pressure-sensitive materials typically fail through the formation of shear bands within which there is accumulation of plastic strain that results in damage and the associated deterioration of material properties [37,40]. This in turn decreases plastic resistance and further localization continues until fracture finally occurs within the shear bands [41]. Such material damage in topology optimization can be directly accounted for by employing coupled elastoplastic damage models, as recently proposed by the authors [31]. The coupled damage models have internal damage variables along with the plastic internal variables to describe the evolution of microstructure, and rate equations for internal damage variables are simultaneously solved with rate equations for plastic internal variables, resulting in increased computational effort for these complex constitutive models. As an alternative approach, which is utilized in this study, uncoupled damage models can be used. Uncoupled models employ a damage criterion, which is satisfied at failure initiation; however, unlike coupled elastoplastic damage models, they do not simulate continuous damage evolution [42,43]. Uncoupled models are easy to calibrate and can be used when the emphasis is on the prediction of final failure rather than on the evolution of damage before failure. Beginning with the study of Johnson and Cook [44], a number of uncoupled damage criteria have been developed to define limiting conditions for fracture initiation in terms of stress-states and accumulated plastic strain. These criteria have been used to predict failure in materials due to either ductile fracture, shear fracture or both [45–47]. In order to limit damage localization in optimal topology designs while still ensuring high plastic work absorption capacity, these fracture criteria can be invoked as constraints during the topology optimization process.

In this study, a density-based topology optimization framework for the design of energy absorbing structures with pressure-dependent yield behavior is proposed wherein the plastic work is maximized. In addition, the accumulation of plastic strain and the associated damage is managed through the use of damage constraints. The Drucker-Prager plasticity model is employed to capture the pressure sensitive yield behavior and a shear fracture criterion is used to define the accumulation of damage that occurs in pressure sensitive materials. An adjoint method is presented to calculate the path-dependent sensitivities of the objective and constraint functions. The sensitivity analysis is also verified using the central difference method. A number of numerical examples are considered to illustrate the ability of the proposed topology optimization framework to design energy absorbing structures. In particular, optimization studies are carried out to investigate the influence of pressure sensitivity in the yield function on optimal topologies and to demonstrate the effectiveness of damage constraints to distribute the material in a more efficient way. The paper is organized as follows: Section 2 gives the governing equations for equilibrium along with their finite element discretization while Section 3 details the Drucker-Prager plasticity model and the shear fracture criterion employed. Section 4 presents the material interpolations used as well as the optimization problems considered in this study. Section 5 gives the accurate path-dependent adjoint sensitivity analysis used in this study and Section 6 presents the numerical examples. Finally, Section 7 offers conclusions.

## 2. Governing equations and discretization

### 2.1. Strong form of initial boundary value problem

Consider a body $\mathcal{B}$ occupying an open set $\Omega \subset \mathbb{R}^3$ consisting of material points $\hat{\boldsymbol{x}} \in \mathbb{R}^3$, shown in Fig. 1. The boundary of the body $\partial\Omega$ is considered to be decomposed into the disjoint sets $\partial\Omega_u$ and $\partial\Omega_\sigma$ such that $\partial\Omega = \partial\Omega_u \cup \partial\Omega_\sigma$ and $\partial\Omega_u \cap \partial\Omega_\sigma = \emptyset$. Neumann boundary conditions corresponding to surface tractions are applied to $\partial\Omega_\sigma$ while Dirichlet boundary conditions corresponding to displacements are applied to $\partial\Omega_u$.

The equilibrium of $\mathcal{B}$ is found by solving the following strong form initial boundary value problem with the displacement, stress and body force fields in $\Omega$ denoted as $\boldsymbol{u} \equiv \boldsymbol{u}(\hat{\boldsymbol{x}})$, $\boldsymbol{\sigma} \equiv \boldsymbol{\sigma}(\hat{\boldsymbol{x}})$ and $\boldsymbol{b} \equiv \boldsymbol{b}(\hat{\boldsymbol{x}})$, respectively

$$\begin{aligned}
&\text{Given } \boldsymbol{b}(\hat{\boldsymbol{x}}): \Omega \to \mathbb{R}^3,\ \bar{\boldsymbol{u}}(\hat{\boldsymbol{x}}): \partial\Omega_u \to \mathbb{R}^3,\ \bar{\boldsymbol{t}}(\hat{\boldsymbol{x}}): \partial\Omega_\sigma \to \mathbb{R}^3\\
&\text{Find } \boldsymbol{u}(\hat{\boldsymbol{x}}): \bar{\Omega} \to \mathbb{R}^3 \text{ such that}\\
&\nabla.\, \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0} \quad \text{in } \Omega\\
&\boldsymbol{u} = \bar{\boldsymbol{u}}(\hat{\boldsymbol{x}}) \quad \text{on } \partial\Omega_u\\
&\boldsymbol{t} = \boldsymbol{\sigma}.\, \boldsymbol{n} = \bar{\boldsymbol{t}}(\hat{\boldsymbol{x}}) \quad \text{on } \partial\Omega_\sigma
\end{aligned} \tag{1}$$

where $\bar{\Omega} = \Omega \cup \partial\Omega$ is the closure of $\Omega$. Eq. $(1)_1$ is the balance of linear momentum and $\bar{\boldsymbol{u}}(\hat{\boldsymbol{x}})$ and $\bar{\boldsymbol{t}}(\hat{\boldsymbol{x}})$ are the prescribed boundary displacement and traction fields, respectively. At each point $\hat{\boldsymbol{x}}$ the stress is given as $\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\boldsymbol{\varepsilon})$, where $\boldsymbol{\varepsilon} \overset{\text{def}}{=} \nabla^s \boldsymbol{u}$ is the small strain tensor and the function $\hat{\boldsymbol{\sigma}}(\boldsymbol{\varepsilon})$ depends on the considered constitutive model.

### 2.2. Weak form and finite element discretization

To approximate equilibrium solutions, the weak form of the initial

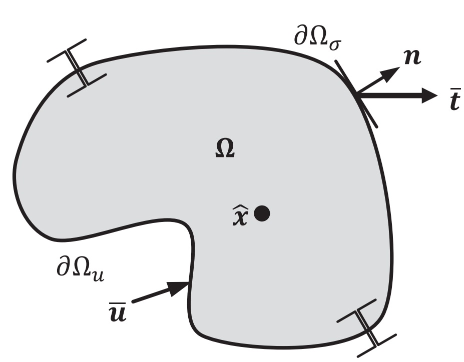


**Fig. 1.** Continuum body with Neumann and Dirichlet boundary conditions.