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An XFEM/level set strategy for simulating the piezoelectric spring-type interfaces with apparent physical background

BingRui Wang[a], JianTao Liu[a,b,*], ShuiTao Gu[c]

[a] School of Mechanical Engineering, Southwest Jiaotong University, Chengdu 610031, China
[b] Laboratoire de Mécanique d'Evry, Université d'Evry, 40 rue du Pelvoux, Évry 91020, France
[c] School of Mechanics and Materials, Hohai University, Nanjing 210098, China

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ABSTRACT

The piezoelectric spring-type interface is widely applied to describe the physical phenomenon that the displacement vector and the electric potential suffer jumps across an interface in a certain intelligent material, while both the normal traction vector and the normal electric displacement stay continuous across the same interface. This work is dedicated to accurately depicting the effects of the physics-based piezoelectric spring-type interfaces in arbitrary shapes and to predicting the effective electroelastic moduli of composites containing such interfaces. To achieve this two-fold goal, a computational approach combining the extended finite element method (XFEM) and the level set method (LSM) is developed and interpreted in detail. The accuracy and convergence performance of the elaborated approach are assessed with a benchmark problem for which the exact analytic solution is derived. Eventually, the approach validated is further utilized to explore the size- and shape-dependent effects induced by these spring-type interfaces on the overall couple-field moduli of fibrous piezoelectric composites.

1. Introduction

Nowadays, piezoelectric materials (PZMs) attract more and more scholars in view of their excellent reciprocal conversion capability between the electrical and mechanical energies, and various contributions have been made to recognizing the intrinsic coupling properties [1–4], identifying the influence factors [5,6] and many others (c.f. [7–10] and the references therein). However, practical applications of the pure PZMs are quite limited ascribed to their inherent brittle and low strength properties. Generally, they are mixed in the form of fibers with one or more other kinds of materials to improve the overall flexibility property and gain better coupling behavior. Such synthesized materials are usually named as the piezoelectric composites (PZCs). In view of their extraordinary electromechanical coupling effects and excellent strength property, PZCs have been widely applied to fabricate the intellectual devices and structures, including the vibration energy harvester, electromechanical transducer, micro-power generator, self-powered wireless sensor, and polyvinylidene fluoride actuator (c.f. [11–16]).

So far, lots of pioneering works have been done on predicting the exact response of PZCs [17–19], estimating the overall material properties [20–22], optimization and design of the intellectual structures [7,14,16] and many others. Among these, a considerable number of studies regard the interface between the PZM and the matrix phases as perfection. However, thin transition layers, which possess totally distinct material property from the bulk ones, often emerge in practical fabrications due to the chemical reaction between different media [7,11,13]. Specifically, micro-defects, such as cracks, voids and so forth, may emerge inside these thin interphases and essentially degrade the overall elasto-electric behavior of PZCs. Commonly, these layers are modeled as imperfect interfaces of null thickness by the asymptotic expansion method [23] to facilitate the theoretical analysis and numerical simulation. In the context of piezoelectric problems, three kinds of imperfect interfaces, namely piezoelectric membrane-type interface, piezoelectric spring-type interface, and piezoelectric general imperfect interface, have been proposed [24–27]. The first one characterizes the phenomenon that both the displacement vector and the electric potential maintain continuous across an interface whereas the normal traction vector and the normal electric displacement suffer jumps across the same interface. Developed first by Gurtin and Murdoch [28] for the elastic problem, this type of models aims mainly to describe the interface energy inside nanocomposites. Concerning the piezoelectric problems, Huang et al. [29] established an interfacial model by taking into account the surface stress effect of piezoelectric

* Corresponding author at: School of Mechanical Engineering, Southwest Jiaotong University, Chengdu 610031, China.
E-mail address: jiantaoliu1982@home.swjtu.edu.cn (J. Liu).

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materials and illustrated that the surface energy plays a dominant role in affecting the electromechanical behavior of piezoelectric nanostructures. Chen [30] further studied the influence of the interface energy characterized by the model in [29] on the effective thermoelectroelastic moduli of a two-phase fiber-reinforced PZC. Mogilevskaya [31] investigated the interacting behavior of multiple nano-inhomogeneities with interface energy by developing a semi-analytical method. Xiao et al. [32,33] derived the closed-form solution of the effective electroelastic moduli of PZCs composed of two and three bulk phases bonded by the membrane-type imperfect interface. Zhang et al. [34] investigated the surface dispersion effect on the propagation of elastic waves inside an infinite piezoelectric plate.

In contrast with the piezoelectric membrane-type interface, piezoelectric spring-type interfaces aim to simulate the phenomena that both the displacement vector and the electric potential suffer jumps across an interface, but the normal traction vector and the normal electric displacement across the same interface are continuous and separately proportional to the displacement vector and the electric potential jumps. Fan et al. [24] proposed a spring-type interface model to account for the electric potential discontinuity within dielectric substances, and related the spring constant k to the number of micro-cracks or -voids along the interface based on the averaging idea of micro-mechanics. Afterwards, they [35] investigated the mechanism of the screw dislocation interacting with the spring-layer imperfect interface. Fang et al. [36] extended the foregoing studies to the piezoelectric problem involving a three-phase cylinder model, and derived the analytic solution of the electroelastic field via the complex variable technique. In Refs. [37,38], Wang et al. first derived the analytic solution of a piezoelectric model consisting of a screw dislocation embedded inside one of the two joined piezoelectric half-planes, which are imperfectly bonded together through a spring-layer interface, and then provided an effective scheme to derive the general solution to 2D Eshelby's problems of an arbitrarily-shaped inclusion embedded inside one of the two imperfectly bonded anisotropic piezoelectric half-planes. Shi et al. [39] derived the analytical effective electroelastic moduli of a three-phase cylindrical PZC model with spring-type interfaces. It is noted that most of the spring-type interface models applied in the aforementioned works are established based on the phenomenological argument.

Piezoelectric general imperfect interfaces stipulate that all physical quantities, i.e. the displacement vector, electric potential, normal traction vector and normal electric displacement, suffer jumps across an interface simultaneously. In particular, the two kinds of imperfect interfaces mentioned above can be retrieved from the general one by separately taking the interface parameters to be highly and lowly conducting. In 2009, Benveniste [25] established a piezoelectric general imperfect interface model by using Taylor expansions to characterize the 3D curved thin interphases between two piezoelectric media. Hereafter, Gu [26] further extended this technique and derived a unified coordinate-free formula of the piezoelectric general interfacial relations. Compared with the one in Ref. [25], the latter model presents the interfacial relations in a more compact form and the derivation is much simpler. Afterwards, they [27] derived the weak formulation of the PZC boundary value problems (BVPs) where the general imperfect interfaces intervene.

The aforementioned interface models and analytic methods are of great use for the piezoelectric BVPs involving specific boundary conditions and interfaces in simple geometries. To handle the PZC BVPs of practical interest, numerical approach becomes an indispensable tool. Among the existing strategies, the extended finite element method (XFEM) [40] combined with the level set method provides an efficient scheme in recognizing and constructing the discontinuities in PZCs. In 2009, Béchet et al. [41] first introduced the XFEM to solve the PZC BVPs involving cracks, and many other developments have been made hereafter (c.f. [42,43] and the references therein). Nanthakumar et al. [44,45] further extended this strategy to detect the flaws inside the piezoelectric structures. Nevertheless, most works reported in the literatures (see e.g. [46–54], etc.) concentrate on the interface effects merely in a certain physical field, say the thermal, the elastic, etc. Precisely, Yvonnet et al. [46] first established a numerical strategy framed within the XFEM and LSM to investigate the highly conducting thermal effect inside heterogeneous materials, and then extended the study to the Kapitza resistance problem in Ref. [53]. Liu et al. [54] extended the XFEM & LSM strategy to account for the general thermal interfacial effect in composites. Concerning the elastic issues, Yvonnet et al. [47] further developed an XFEM/LSM approach in modeling the membrane-type interface effect and studied the size-dependent effective properties of nanocomposites. Meanwhile, Benvenuti in Ref. [48] provided a modeling method via the XFEM for the heterogeneous materials containing the elastodamaging interfaces, which possess similar effect as the elastic spring-type interface. Béchet et al. [49] proposed a new algorithm to define a stable Lagrange multiplier space for applying the stiff interface conditions. Zhu et al. [52] proposed a computational strategy with the aid of the XFEM to simulate the elastic BVPs involving the spring-layer imperfect interfaces. Farsad et al. presented a numerical scheme in [51] by coupling the XFEM and LSM to model the general imperfect interface effects on the mechanical behavior of nanostructures. It is noticed that the general interfacial discontinuities of Ref. [51] are specified artificially. Hereafter, Liu et al. [55] further investigated the effects induced by the physics-based general imperfect interfaces on the overall moduli of particle-reinforced composites. To the best of the authors' knowledge, a few literatures refer to the numerical implementation of imperfect interface problems in couple fields [56–58], and no physics-based piezoelectric spring-type interface model has been numerically implemented framed within the XFEM and LSM, which motivates the generation of the present work. With the aid of a piezoelectric spring-type interface model derived from the practical three-phase configuration, this paper contributes mainly to: (i) elaborating a computational approach to simulate the piezoelectric response of PZCs containing such physics-based interfaces of arbitrary shapes by constructing the relevant governing formulations; (ii) analytically solving a benchmark problem to quantitatively examine the convergence performance and validity of the computational approach; (iii) investigating the effects of the physics-based spring-type interfaces of noncircular cross-section on the overall couple-field moduli of PZCs.

The present paper proceeds as follows. In the section below, a unified governing formula is first introduced for the piezoelectric materials, and then a spring-type interface model with explicit physics background is briefly recalled. In Section 3, both the strong and weak formulations are constructed for the piezoelectric BVPs involving multiple physics-based spring-type interfaces in arbitrary shapes, and the discretization of the weak formulation derived is presented hereafter with the aid of the XFEM and LSM. Section 4 is dedicated to verifying the correctness and efficiency of the elaborated computational approach, and a benchmark problem is designated and analytically solved at first. Finally, the influence of interface parameters, size- and shape-dependent effects on the effective piezoelectric performance of fibrous PZCs is explored in detail in Section 6, and a few concluding remarks are obtained.

2. A piezoelectric spring-type interface model with apparent physical background

We are here interested in the electromechanical coupling behavior of the PZCs reinforced by the fibrous PZMs, see e.g. Fig. 1. The whole 3D domain Ω is bounded by the external surface $\partial\Omega$ ($\partial\Omega = \partial\Omega_w \cup \partial\Omega_t$) with its outward unit normal vector symbolized by $\boldsymbol{n}^{(M)}$. Inside Ω, m reinforcement PZM fibers separately denoted as $\Omega^{(r)}$ ($r = 1, 2, \ldots, m$) are embedded in a matrix phase $\Omega^{(M)}$. Specifically, these fibers are assumed to be straight and infinitely extended along its longitudinal direction, i.e. the z-axis. The interface between $\Omega^{(M)}$ and $\Omega^{(r)}$ is specified as $\Gamma^{(r)}$,