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Local multiaxial fatigue damage estimation for structures under random vibrations

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ABSTRACT

Many structures are subjected to random vibrations and those vibrations can cause a fatigue damage. So, to estimate the live time of the structure, we need to applied the fatigue multiaxial criteria at each point. Many studies demonstrate that the Sines criterion seems to give the best evaluation of the fatigue damage. But the application of this criterion need a high computing time if it's evaluated in the time domain. In this paper, we present a new methodology of calculation of the Local multiaxial fatigue damage, based on the formulation of the Sines criterion developed in the frequency domain. Each parameter of the Sines criterion will be calculated from the Power Spectral Densities of the stresses at each point of the structure. A finite element example is then used, at the end of this paper, to illustrate the application of the proposed strategy of calculation of the fatigue damage.

1. Introduction

In the last few decade, many studies have been elaborated, in order to estimate the life time of a structure which is subject to random vibrations. For multiaxial fatigue case, there are many criteria that calculate the fatigue damage. Such a criterion can be mathematically represented by:

$$g(S_{i,j}(t),\ \mathrm{T}) \le 1 \tag{1}$$

Where $S_{i,j}(t)$ is the stress tensor at any given location in the structure and for any time $t \in [0,T]$. A fatigue crack can appear if the inequality (1) is not satisfied at any point of the structure.

In order to minimize the computing time, the fatigue damage analyses will be done in the frequency domain. This can be seen in many works such as Pitoiset [1,2] which defined a formulation of Matake's criterion and Crossland's criterion in frequency domain. According to Weber et al. [3] and Wu et al. [4], the Sines criterion seems to give the best evaluation of damage subject to random stresses. Then, a formulation of Sines criterion in the frequency domain will be defined.

Let's consider a linear structure subject to stationary ergodic Gaussian loads. According to Pitoiset [5], the crack will not initiate before a certain number of repetitions, Ne, of the periodic random load of duration, T, if the average of the damage does not exceed the fatigue limit.

$$E\left[g(S_{i,j}(t),\mathrm{T})\right] \le 1 \tag{2}$$

Then, to calculate the average of the damage in the frequency domain, we need to calculate the Power Spectral Densities (PSD) of the stresses for all point in the structure.

Also, there are several studies in reliability and optimization for structures under random vibrations taking into account the fatigue damage [6–11].

In the following sections, the damage calculation methodology in the frequency domain by the Sines criterion is presented and discussed. This methodology will be applied next on the same example treated by Pitoiset [2]. Whereas, he used the Matake's criterion and crossland's criterion on a simple steel to determine the fatigue damage. This example will allow us to show the advantage of the Sines criterion in the frequency domain for treating complex structures.

2. Random vibration

2.1. Random process

A random vibration is a motion which is non-deterministic. It's mean that future behavior cannot be precisely predicted. Let's suppose that we record a parameter characterizing a physical phenomenon n times. The set of all these functions {f (t)} is called random process. We can calculate for this process the mean value (Eq. (3)), the variance (Eq. (4)), the autocorrelation function (Eq. (5)) and the covariance

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Matrix (Eq. (6)).

$$E[f(t_1)] = \int_{-\infty}^{+\infty} f(t_1).\ p(f(t_1))df \quad (3)$$

with $p(f(t_1))$ is the probability density of $f(t)$.

$$\sigma_f^2 = E[f^2]-(E[f])^2 \quad (4)$$

$$R_f(t_1, t_1+\tau) = E[(f(t_1)).\ (f(t_1+\tau))^T] \quad (5)$$

$$\Sigma_f(t_1, t_1+\tau) = E[(f(t_1)-\overline{f(t_1)}).\ (f(t_1+\tau) - \overline{f(t_1+\tau)})^T] \quad (6)$$

It's called a Gaussian stationary ergodic process if all its statistical properties are invariable with time and if its overall statistical properties are equal to the temporal properties of any taken sample.

2.2. Power spectral density

The power spectral density is the Fourier transformation of the autocorrelation function (Wiener-Kintchine theorem). The Eq. (7) presents the PSD of a signal f(t)

$$\Phi_f(f) = \int_{-\infty}^{+\infty} R_\ell(\tau)e^{-j2\pi f\tau}d\tau \quad (7)$$

The PSD determines the distribution of the process energy in the frequency domain. Then, we can characterize a random vibration by its PSD.

If the process f(t) has units of meters, and the frequency f (the independent variable) has units of Hz then the units of the PSD is m²/Hz. Alternatively, if the angular frequency $\omega=2\pi f$ is the independent variable, then the units of the PSD is m²/(rad/s). So, to convert between frequencies f and ω, the value of the PSD needs to be scaled as well [12]:

$$\Phi_\ell(\omega)d\omega = 2\pi\Phi_\ell(f)df \quad (8)$$

2.3. Statistic proprieties of Gaussian random process

A spectral moment of a random process is defined by:

$$m_i = \int_{-\infty}^{+\infty} |f|^i\Phi_\ell(f)df \quad (9)$$

These moments contain important information about the process. They allow to evaluate many characteristics of the process such as the variances of this process and its derivatives.

From the spectral moments calculated previously, an analytical formulation can be developed to study the statistical properties of random Gaussian processes such that the average number of passage through a level b with a positive slope:

$$N_b = \sqrt{\frac{m_2}{m_0}}e^{-\frac{b}{2m_0}} \quad (10)$$

It can be deduced from Eq. (10) the number of up-crossings of level zero (Eq. (11)) and the average of the number of maxima (Eq. (12)) [13].

$$N_0 = \sqrt{\frac{m_2}{m_0}} \quad (11)$$

$$N_p = \sqrt{\frac{m_4}{m_2}} \quad (12)$$

2.4. Response of a linear system under random vibration

Let's suppose that f (t) is the input signal of a linear system, g (τ) is the impulsion response, and u (t) is the output signal.

We can calculate for this system the mean value E[s(t)] (Eq. (13)) and the PSD Φ_s (Eq. (14)) of the Stresses. Thus, the stresses PSD is calculated from the excitation input PSD Φ_f. [14]

$$E[s(t)] = HBK^{-1}E[f(t)] \quad (13)$$

$$\Phi_s(\omega) = HB|G(\omega)|^2\Phi_f(\omega)B^TH^T \quad (14)$$

With H is the elastic coefficient matrix and B is the interpolation functions gradient matrix.

In the case of plane stress, the PSD matrix can be written as [15]:

$$\Phi_s = \begin{bmatrix} \Phi_{s_{xx}s_{xx}} & \Phi_{s_{xx}s_{yy}} & \Phi_{s_{xx}s_{xy}} \\ \Phi_{s_{yy}s_{xx}} & \Phi_{s_{yy}s_{yy}} & \Phi_{s_{yy}s_{xy}} \\ \Phi_{s_{xy}s_{xx}} & \Phi_{s_{xy}s_{yy}} & \Phi_{s_{xy}s_{xy}} \end{bmatrix} \quad (15)$$

3. Multiaxial fatigue damage analysis

3.1. Formulations and developments

The main purpose of multiaxial fatigue criteria is to determine the location of the critical point in a structure. It can be assumed, by many authors, that the crack initiation is governed by the second invariant of the deviator stress tensor [16,17]. This can be seen in the Crossland's criterion (Eq. (16)) and Sines Criterion (Eq. (17)) where $J_{2,a}$ is the second invariant of the deviator stress tensor.

$$D_{Crossland} = \frac{\sqrt{J_{2,a}} + \alpha \max_{0\le t\le T}\{p(t)\}}{\beta} \le 1 \quad (16)$$

$$D_{Sines} = \frac{\sqrt{J_{2,a}} + \alpha E[\mathrm{p(t)}]}{\beta} \le 1 \quad (17)$$

Where p(t) is hydrostatic pressure which is determined by the normal stresses components S_{xx}, $S_{yy}$and S_{zz} of the stress tensor S_{ij} (Eq. (18)) and α, β are constants related with the material. For the Sines criterion, they are defined by the endurance limit in alternating traction f_{-1}, the endurance limit in alternating torsion t_{-1} and the breaking strength R_m.(Eqs. (19 and 20)).

$$\mathrm{p(t)} = \frac{1}{3}Trace\{S_{ij}\}\frac{1}{3}(S_{xx}+S_{yy}+S_{zz}) \quad (18)$$

$$\alpha = \frac{3t_{-1}(R_m+f_{-1})}{f_{-1}R_m} - \sqrt{6} \quad (19)$$

$$\beta = t_{-1} \quad (20)$$

It can be noted that the Crossland's criterion need the calculation of maximum of the hydrostatic pressure while the Sines criterion uses it's mean. This reduces the calculation time of the Damage if it's calculated by the Sines criterion and have almost the same result such as the Crossland's criterion [18].

Let's consider now a linear structure subject to stationary ergodic Gaussian loads. According to Eq. (2), the crack will not initiate before Ne repetitions of the periodic random load of duration T if the average of the damage should not exceed the fatigue limit. The Sines criterion is defined then by:

$$E[D_{Sines}] = \frac{E[\sqrt{J_{2,a}}] + \alpha E[\mathrm{p(t)}]}{\beta} \le 1 \quad (21)$$

The problem now is the determination of the average of the shear stress amplitude $E[\sqrt{J_{2,a}}]$.

In the time domain $\sqrt{J_{2,a}}$ is defined by the Eq. (22).

$$\sqrt{J_{2,a}} = \frac{1}{\sqrt{3}}\max_{0\le t\le T}|s_c(t)| \quad (22)$$

Here $s_c(t)$ is the Von Mises Constrain. The problem here is that $s_c(t)$