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Numerical analysis of 3-D solids and composite structures by an enhanced 8-node hexahedral element

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ABSTRACT

A novel enhanced 8-node hexahedral element based on the concept of consecutive-interpolation procedure (CIP) is developed for the analysis of three-dimensional (3-D) linear solids and composite structures. The developed element is named as CHH8 for the brevity. The patch test, locking test, free vibration and heat transfer problems for simple and complex 3-D solids are analyzed. We also consider a 3-D sandwich beam with a core layer made of polyurethane foam and two skin layers made of orthotropic carbon/epoxy composite. The significant difference of the CIP-based approach, as compared with traditional finite element method, is the integration of averaged nodal gradients into interpolation process, representing as a nonlocal feature. With these additional terms, the proposed CHH8 element is able to produce higher accurate approximation of physical fields and smooth gradient fields which are continuous across element boundaries. More importantly, these advantages can be achieved without increasing the problem size, because the degrees of freedom still remain the nodal values. Details about the proposed CHH8 element are presented. To show the accuracy of the developed CHH8 element, numerical examples of 3-D solids and composite structures with simple and complex configurations are considered, and the obtained results are then compared with reference solutions derived from analytical, other numerical methods and experimental data.

1. Introduction

Finite element method (FEM) is well-estabilished and is known as one of the most popular numerical methods for engineering problems. However, the FEM inherently owns several shortcomings as mentioned in [1,2]. One great limitation is that the shape functions used in FEM are C^0-continuous, thus the nodal gradient fields, e.g., the temperature gradients of heat transfer problems or strain/stress fields of mechanical problems, are non-physically discontinuous across element boundaries. In practice, this issue is usually required to be treated during post-processing.

The boundary element method (BEM) [3,4] suggests to solve boundary value problems without domain discretization. Advantages of BEM has been shown for some specific problem, such as crack modeling. However extraction of data points inside the problem domain is quite challenging. Furthermore, fundamental solutions are often required, which is not a trivial task in practice and thus limits the application of the method.

New or improved numerical methods for engineering problems have been one of major subjects to the scientific community, and various alternatives have thus been previously introduced in literature. A class of meshfree methods follow a different concept, in which the problem domain is represented only by scattered nodes, including one set of nodes inside the problem domain and another set of nodes on its domain boundary. The solution steps of meshless methods are generally similar, although not analogous, to those of FEM. Different versions of meshfree methods have been introduced in the literature including element-free Galerkin (EFG) method [5], reproducing kernel particle method (RKPM) [6], moving Kriging interpolation method [7], meshless local Petrov-Galerkin (MLPG) method [8], point interpolation method (PIM) [9] and radial point interpolation method (RPIM) [10]. Most of the meshfree methods, except the PIM and RPIM, do not possess the Kronecker-delta property, and thus the boundary condition cannot directly be imposed as that of the FEM, and are usually treated with the aid of Lagragian multipliers or penalty method [5]. Meshfree methods have successfully been applied to solve a wide range of engineering problems, e.g., see [11,7] and references therein.

Isogeometric Analysis (IGA) [12,13] is an effective numerical method, which employs non-uniform rational B-spline (NURBS) as

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basis functions, originally used in Computer-Aided Design (CAD), for both representing geometry and approximating field variables. The advantages of IGA is the full control in shape function order and continuity. The disadvanges are the complexity in implementation and difficulty in treatment of boundary conditions, as NURBS basis functions in general do not possess the Kronecker-delta property.

Improvement of FEM draws much attention, with the aim to keep the advantages of FEM while reduces its disadvantages. The recently proposed consecutive-interpolation procedure (CIP) approach is among the latest improvements focusing on providing results in which gradient fields are continuous across element boundaries, which is physically favorable. The approach was formulated for three-node triangular element (CT3), and four-node quadrilateral element (CQ4) for two-dimensional linear elastic problems [2,14,15], and for two-dimensional linear elastic fracture mechanics [16,17]. More recently, Nguyen et al. [18] applied the CIP to the 4-node tetrahedral element to form a new element named as CTH4, which is devoted to 3-D heat transfer problems.

The major motivation of the CIP-based elements is to make the trial solution and its derivatives continuous across inter-element boundaries [2,15]. This not only can improve the accuracy of the computed gradients of the solution, but also avoid using smoothing techniques, which is often used in the post-processing step. Previous numerical solutions of elastostatic, elastodynamic and fracture problems reported in [2,14–17] have shown that the CIP-based elements offer, for the same number of degrees of freedom, higher accuracy of the solutions and better convergence than those for the conventional T3 and Q4 elements. Moreover, smoothed stresses at nodes can also be obtained without using smoothing operations. As pointed out in [2,15] that the shape functions for the CIP-based elements are linearly independent, satisfy the partition-of-unity, possess the Kronecker-delta function property, and alleviate the volumetric locking issue for incompressible materials. As the CIP does not alter the usual procedure of finite element analysis, the CIP-based elements can easily be implemented in existing FEM codes. Nevertheless, the main desirable features of the CIP-based elements have already been discussed and pointed out in our previous published papers, e.g., see [2,15–18], and also [14]. Therefore, we do not repeat them here, and curious readers may refer to the given references for more detail.

The main objective of this paper is to develop a novel effective 3-D element, which integrates the CIP into the 8-node hexahedral element (named as CHH8) to solve 3-D linear elasticity and composite structures. The natural frequency and heat transfer of 3-D solids are analyzed. A 3-D sandwich beam with a core layer made of polyurethane foam and two skin layers made of orthotropic carbon/epoxy composite is also analyzed. It is expected that the novel CHH8 element could yield and provide better accuracy than the conventional hexahedral element (HH8), tetrahedral element (TH4), even CTH4 element [18]. Another purpose of this paper is to verify the general formulation described in [18] to this new CHH8 element. All the numerical results computed by using the developed CHH8 element are validated with respect to reference solutions, which are derived from analytical results, other numerical methods and experimental data.

The rest of this paper is structured as follows. After the introduction, formulation of the novel 3-D CHH8 element is presented in detail. The weak form of linear elastic problems and their numerical results are given in Section 3. The analysis of heat transfer problems is presented in Section 4. Application to composite structures is presented in Section 5. Our conclusions drawn from this study are given in Section 6.

2. Formulation of consecutive-interpolation 8-node hexahedral element

2.1. Preliminaries to the consecutive-interpolation (CIP) technique

Let us consider a 3-D body in the domain Ω bounded by the boundary Γ. A function $u(\mathbf{x})$ defined in Ω is approximated by the consecutive-interpolation (CIP) scheme as

$$u(\mathbf{x}) \approx \tilde{u}(\mathbf{x}) = \sum_{I=1}^{n} (\phi_I u^{[I]} + \phi_{Ix}\overline{u}_{,x}^{[I]} + \phi_{Iy}\overline{u}_{,y}^{[I]} + \phi_{Iz}\overline{u}_{,z}^{[I]}), \tag{1}$$

where n is the number of nodes and $u^{[I]}$ is the value of function $u(\mathbf{x})$ evaluated at node I (global numbering) by the finite element interpolation

$$u^{[I]} = u(\mathbf{x}_I) = \sum_{l=1}^{n} N_l(\mathbf{x}_I)\hat{u}_l = \mathbf{N}^{[I]}\hat{\mathbf{u}} \tag{2}$$

In comparison with standard FEM, the additional values $\overline{u}_{,x}^{[I]}$, $\overline{u}_{,y}^{[I]}$, $\overline{u}_{,z}^{[I]}$ are the averaged nodal gradient of $u(\mathbf{x})$ evaluated at node I, corresponding to x, y and z direction, respectively. The first order derivative of $u_{,x}^{[e]}(\mathbf{x}_I)$ evaluated at node I (global numbering) within an element e, can be written by finite element interpolation as follows:

$$u_{,x}^{[e]}(\mathbf{x}_I) = \sum_{l=1}^{ne} N_{l,x}\hat{u}_l = \mathbf{N}_{,x}^{[I][e]}\hat{\mathbf{u}}, \tag{3}$$

with ne being the number of nodes within element e. With the nodal gradients $u_{,x}^{[e]}(\mathbf{x}_I)$ ready, for all the elements $e \in S_I$ that share the node I, the averaged value $\overline{u}_{,x}^{[I]}$ can then be calculated using weighted averaging by

$$\overline{u}_{,x}^{[I]} = \sum_{e\in S_I} (w_e \mathbf{N}_{,x}^{[I][e]})\hat{u}_l = \overline{\mathbf{N}}_{,x}^{[I]}\hat{\mathbf{u}} \tag{4}$$

Here, the weights w_e are defined by the ratio of the volume of element e and the total volume of the set S_I. Similarly, the averaged nodal derivatives with respect to y and z direction, i.e. $\overline{u}_{,y}^{[I]}$ and $\overline{u}_{,z}^{[I]}$, can be computed in such a way. In order to maintain the Kronecker-delta property, the auxiliary functions ϕ, ϕ_{Ix}, ϕ_{Iy} and ϕ_{Iz} in Eq. (1) have to be determined for each type of element and must satisfy the following conditions

$$\begin{aligned}
&\phi_I(x_J) = \delta_{IJ}, \quad \phi_{I,x}(x_J) = 0, \quad \phi_{I,y}(x_J) = 0, \quad \phi_{I,z}(x_J) = 0\\
&\phi_{Ix}(x_J) = 0, \quad \phi_{Ix,x}(x_J) = \delta_{IJ}, \quad \phi_{Ix,y}(x_J) = 0, \quad \phi_{Ix,z}(x_J) = 0\\
&\phi_{Iy}(x_J) = 0, \quad \phi_{Iy,x}(x_J) = 0, \quad \phi_{Iy,y}(x_J) = \delta_{IJ}, \quad \phi_{Iy,z}(x_J) = 0\\
&\phi_{Iz}(x_J) = 0, \quad \phi_{Iz,x}(x_J) = 0, \quad \phi_{Iz,y}(x_J) = 0, \quad \phi_{Iz,z}(x_J) = \delta_{IJ}
\end{aligned} \tag{5}$$

Eq. (1) can then be rewritten to approximate the value of function u at arbitrary point $\mathbf{x}$ located within an element e

$$u(\mathbf{x}) \approx \tilde{u}(\mathbf{x}) = \sum_{I=1}^{n} \left(\phi_I \mathbf{N}^{[I]} + \phi_{Ix}\overline{\mathbf{N}}_{,x}^{[I]} + \phi_{Iy}\overline{\mathbf{N}}_{,y}^{[I]} + \phi_{Iz}\overline{\mathbf{N}}_{,z}^{[I]}\right)\hat{\mathbf{u}} = \mathbf{R}\hat{\mathbf{u}}, \tag{6}$$

where

$$\mathbf{R} = \sum_{I=1}^{n} \left(\phi_I \mathbf{N}^{[I]} + \phi_{Ix}\overline{\mathbf{N}}_{,x}^{[I]} + \phi_{Iy}\overline{\mathbf{N}}_{,y}^{[I]} + \phi_{Iz}\overline{\mathbf{N}}_{,z}^{[I]}\right) \tag{7}$$

The CIP-based shape function associated with node l, R_l, can then be explicitly written by

$$R_l(\mathbf{x}) = \sum_{I=1}^{n} [\phi_I(\mathbf{x})N_l(\mathbf{x}_I) + \phi_{Ix}(\mathbf{x})\overline{N}_{l,x}(\mathbf{x}_I) + \phi_{Iy}(\mathbf{x})\overline{N}_{l,y}(\mathbf{x}_I) + \phi_{Iz}(\mathbf{x})\overline{N}_{l,z}(\mathbf{x}_I)] \tag{8}$$

It is noticed that there might exist more than one set of auxiliary functions for any given single arbitrary type of finite element. In the reference works, e.g., see [14,2,16], only the polynomials-based auxiliary functions are given, and the set of auxiliary functions are developed specifically for each element type, e.g. three-node triangular element and four-node quadrilateral element. Recently, the work reported by Nguyen et al. [18] however proposed a general formulation for polynomial-type auxiliary functions, defined locally in each element e. Consequently, the auxiliary functions at the local i^{th} node of element e are thus calculated by