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A Bi-directional Evolutionary Structural Optimisation algorithm with an added connectivity constraint

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ABSTRACT

This paper proposes the introduction of a connectivity constraint in the Bi-directional Evolutionary Structural Optimisation (BESO) method, which avoids the possibility of arriving at highly non-optimal local optima. By developing a constraint that looks at the usefulness of complete members, rather than just elements, local optima are shown to be avoided. This problem, which affects both evolutionary and discrete optimisation techniques, has divided the optimisation community and resulted in significant discussion. This discussion has led to the development of what is now known in the literature as the Zhou-Rozvany (Z-R) problem. After analysing previous attempts at solving this problem, an updated formulation for the convergence criteria of the proposed BESO algorithm is presented. The convergence of the sequence is calculated by the structure's ability to safely carry the applied loads without breaking the constraints. The Z-R problem is solved for both stress minimisation and minimum compliance, further highlighting the flexibility of the proposed formulation. Finally, this paper aims to give some new insights into the uniqueness of the Z-R problem and to discuss the reasons for which discrete methods struggle to find suitable global optima.

1. Introduction

Evolutionary algorithms are characterised by ease of application, greater robustness and lower complexity compared with most of the classical methods used for optimisation [10]. In light of these benefits, evolutionary methods have been criticised for their lack of algorithmic convergence and selection of appropriate stopping criteria [29]. One such criticism of the ESO/BESO method showed that hard-kill methods could arrive at highly non-optimal designs, when applied to a beam-tie problem [38]. This test case became known as the Zhou-Rozvany problem and it is the aim of this article to develop a BESO algorithm that avoids these convergence issues and is capable of producing optimal designs for this problem.

Zhou and Rozvany show that ESO's rejection scheme can result in highly non-optimal designs and that the readmission scheme used in the BESO algorithm is unable to correct this failure [38]. Zhou and Rozvany blame this shortfall of the algorithm on the inability of the sensitivity numbers to give a good estimate of the change in the objective function. This is due to the heuristic nature of the algorithm, i.e. the thickness of the elements are discrete [0 1] with no intermediate values. Failure of the sensitivity number occurs when its value varies significantly with respect to its normalised density [38]. Since evolutionary methods are discrete this change is not identified and the element is never readmitted. Nevertheless, the merits of evolutionary algorithms have been recognised by the gradient-based optimisation community and improvements have been suggested. Rozvany and Querin suggested that the sensitivity numbers given in the prior iteration should be compared with the actual finite change of the objective function [23]. If a large discrepancy is obtained the element is kept or banned from being removed. Zhou pointed out at the WCSMO-7 congress that this proposed improvement may become uneconomical for the very large systems used in practice [21]. Rozvany gives an example of 100 rejected elements out of 100,000 total elements, resulting in 100 checks that have to be made in each iteration [21]. This would significantly increase the computational cost of the algorithm, nullifying one of the benefits of evolutionary methods.

From this early criticism a new method, termed Sequential Rejections and Admissions (SERA), was developed by Rozvany and Querin [24]. The sensitivity numbers are calculated from the Lagrange multipliers, as in the case of gradient-based methods [39,40]. SERA also introduces the concept of virtual material, where instead of completely removing an element an infinitesimal density/thickness is assigned. The success of this method was demonstrated at the WCSMO-4 congress [22], but with the implementation of virtual material, the approach is no longer a hard-kill topology optimisation method. Furthermore, the algorithm still suffers from an inaccurate

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sensitivity analysis and is unable to solve the Z-R problem. Nevertheless, such improvements have been applied successfully to multiconstraint problems [25] and flexible structures [1].

Another solution is to use a finer finite element mesh in the analysis [5,6,8,19,41,15]. This goes against some of the algorithm's merits, such as low Finite Element Analysis (FEA) time [34], however it was shown to lead to optimal solutions for the Z-R problem in its original formulation [9]. In spite of this success, Rozvany showed in a recent critical review of established topology optimisation methods that, for any mesh refinement, the ESO algorithm would still produce non-optimal results to the Z-R problem if the magnitude of the loads were increased [21]. Huang and Xie show that the solution to the original Z-R problem obtained by the ESO/BESO method is a highly inefficient local optimum, instead of a nonoptimum [10]. They demonstrated that, if the penalty number was increased to $p \geq 3.1$, then the SIMP algorithm would arrive at the same local optimum as the ESO/BESO methods. Therefore one cannot dismiss an optimisation method that arrives at a local optimum for a non-convex problem. To avoid the possibility of arriving at local optima, Huang and Xie suggest checking the boundary and loading conditions after every design iteration [11]. If after a certain design iteration the boundary and/or loading conditions have changed, the sequence cannot continue until corrective measures have been taken. However, Rozvany showed that this procedure can cause computational complications [21]. Furthermore, an initial problem may be given with many possible supports, some of which are uneconomical to use. One of the aims of topology optimisation for multiple support problems, such as the Z-R problem, is to select the supports that should be used. This would not be possible if all boundary conditions were frozen.

Recently, Stolpe and Bendsoe considered the minimum compliance variation of the Z-R problem and developed a nonlinear branch and cut algorithm to solve for global optima [33]. They showed several optimal designs for varying volume constraints; the only example of this kind of heuristic algorithm solving this class of problem. Ghabraie examined the evolutionary algorithm and its application to the Z-R problem [7]. He proposed a problem statement for ESO and performed an accurate sensitivity analysis, mathematically justifying the algorithm. He showed that by including the higher order terms of the Taylor expansion in the sensitivity analysis the ESO method is not prone to the problems proposed by Zhou and Rozvany [38]. However, even the revised ESO algorithm is unable to solve the Z-R problem, suggesting that the reason for this is ESO's inherent unidirectional approach [7]. He concluded that the ESO method should only be used on a very specific class of problems, where the constraints demand a unidirectional approach to final solutions. He also suggested that the BESO method is not prone to this shortcoming, but did not apply this approach to the Z-R problem.

2. Research gaps

It has been demonstrated in the literature review (Section 1) that the ESO/BESO algorithm can suffer from convergence issues and an inaccurate sensitivity analysis. This has mainly been shown through the Z-R problem where the current ESO/BESO algorithms struggle to find a suitable solution, however:

- While it is known that the sensitivity analysis of ESO/BESO algorithms can become inaccurate it is not known when. This article shows that by removing one element from the domain the nature of the problem is changed. The BESO algorithm tries to solve this new problem and never returns to the old problem. However, if this change in nature is picked up then the problem is never changed and the BESO algorithm is able to solve the original problem.
- Prior researches have shown that refining the mesh can provide more suitable topologies to the problem. However, this is at a great expense in computational time. In this paper refined meshes are analysed to show that the updates to the algorithm can find the same final topologies at a much lower computational expense. Thus, maintaining the original benefits of ESO/BESO algorithms.
- One prior method for avoiding arriving at highly inefficient local optima is the so called "*freezing*" approach suggested by [9]. While this approach is suitable for solving the Zhou-Rozvany problem with BESO methods, [21] points out that this method would fail to determine which supports would be economical to use for multiple support problems. Furthermore, [7] noted that one of the shortcomings of this method is that it can also fail on statically determinate problems, where there is no change in boundary or loading conditions. Therefore, in this paper, a statically determinate problem suggested by [7] is solved to show the improvements of the presented approach over the freezing approach.
- Finally, a recent study [33] has provided a range of different optima for varying volume fractions to the Z-R problem to provide a benchmark for new or modified algorithms. In this study the modified BESO algorithm is verified against some of these optima.

The aim of this work is to develop a connectivity constraint to embed inside the BESO method so that the topology can arrive at optimal solutions. This updated technique is used to solve the Z-R problem, as it is special in its formulation in that the solution can become two completely different problems by the removal of a single element. The Z-R problem will be solved in its different conceptions, i.e. stress and compliance, as well as different volume fractions and grid resolutions, that have been developed over the years by the topology optimisation community. The updated technique uses a similar approach to that suggested by Rozvany and Querin, where the sensitivity numbers given in the prior iteration should be compared with the actual finite change of the objective function [23], but the algorithm presented in this article only performs this check when the connectivity of a member is broken; it assesses whether or not the member is economical, thus improving computational efficiency. The solution suggested by Rozvany and Querin [23] was never implemented due to its excessive computational burden [21]. The new connectivity constraint works by checking whether the objective function of the structure is increased above a predefined tolerance, only for the elements that do not satisfy connectivity. If this change is above the tolerance then the elements are readmitted to reconnect the structure. Otherwise, the complete structural member, or elements which do not satisfy connectivity, are removed.

3. Theoretical analysis

The algorithm used in this article is a hard-kill BESO method with a novel connectivity criterion. The aim of this work is to fundamentally understand why ESO/BESO algorithms suffer from convergence problems and how this can be avoided in a manner that does not add a significant computational burden, using the Z-R problem as a test case. Furthermore, elements with very small positive densities, such as those found in soft-kill BESO, a discrete method with a non-zero density material defined for void elements, and SIMP algorithms, a continuous method with densities varying between a very small positive number and 1, may cause the global stiffness matrix to become ill-conditioned, particularly for a nonlinear structure [17]. For these reasons the hard-kill BESO method may be preferable, though it still suffers from computational difficulties due to its discrete nature as outlined in Section 1.

3.1. Hard-Kill Bi-directional Evolutionary Structural Optimisation (BESO)

The BESO method, in its simplest form, determines the optimal topology of a structure according to the relative ranking of its sensitivity numbers. The BESO method, unlike the ESO method, allows

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