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Universal meshes for a branched crack

Yongxing Shen*, Can Wu, Yang Wan

State Key Laboratory of Metal Matrix Composites, University of Michigan-Shanghai Jiao Tong University Joint Institute, Shanghai Jiao Tong University, Shanghai, China

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ABSTRACT

We introduce an efficient approach to obtain conforming meshes for evolving branched cracks immersed in a fixed background mesh. The proposed approach is built on universal meshes (UM) proposed in Rangarajan et al. [34] which is able to construct conforming triangulations for a propagating simple crack. The UM functions by projecting certain nodes of the background mesh onto the crack and simultaneously relaxing their neighboring nodes to improve the quality of the resulting triangular mesh. The essence of the generalization to a branched crack is to determine which side of each branch to select nodes to move to the crack path. The choice is based on the consideration of minimizing mesh distortion. For the case of multiple junctions, we take into account the constraint that the nodes to be moved to the same crack branch must be on the same side of that branch. The proposed method inherits the main advantages of UM, including small perturbation to a fixed background mesh for a family of evolving cracks with no *a priori* conformity requirements. This advantage saves computational time compared with a brute-force mesh generation step. Numerical examples with one or multiple triple or quadruple junctions are provided.

1. Introduction

Dynamic crack analysis is essential in applications such as impact and explosion. Very often, an initially simple crack will bifurcate into two or more branches. Another scenario when bifurcation may occur is when the cracked material exhibits certain heterogeneity. Experimental and theoretical study of such phenomena has been an active field of research [1–9]. A typical example is hydraulic fracturing: when the hydraulic fracture intersects the natural fissures or layers, it may bifurcate into multiple branches, resulting in a complex fracture network [10].

Numerical analysis of crack propagation with the possibility of bifurcation allows gaining more insights of the process and more information for engineering designs. Such numerical methods can be roughly classified into two families: One explicitly taking into account the discrete crack paths, and the other adopting a smeared description of the cracks.

The smeared description of cracks is appealing since apparently the crack path evolution is obtained for free, without extra criteria for nucleation and bifurcation other than the evolution of the primary fields. Phase field methods for fracture are a typical kind of methods in this family [11–16], with [17] focused on branching. One of the drawbacks of such methods is, due to the *a priori* unknown crack path, the need to either employ a very fine mesh or an adaptively refined mesh [18,19]. The gradient damage models [20] are another types of models of this kind.

In contrast, the discrete crack approaches generally simulate the crack evolution by obtaining the displacement, velocity and stress fields, calculating the crack increment and then updating the spacial discretization for the evolving crack. This requires two ingredients: An efficient solver for the elastic or elastoplastic field, and a branching criterion (including the critical crack speed and the directions of the new branches) such as that proposed by Katzav et al. [21]. For the first ingredient, distinction can be made on whether the spatial discretization needs to conform to the evolving cracked domain.

Employing a crack-independent mesh, the extended finite element method (XFEM) [22–24] approximates the displacement field via the introduction of Heaviside functions and leading terms of the near-tip asymptotic expansion. Nevertheless, XFEMs require processing elements divided by arbitrary crack paths, leading to complicated geometric programming, involved numerical integration, and possibly ill-conditioned stiffness matrix due to arbitrarily small pieces cut by the crack. Such disadvantages are overcome by various modifications of the method; however, if a conforming mesh is adopted, the aforementioned issues will disappear.

Several approaches can be used to obtain a conforming mesh for a propagating crack, or more generally, an evolving domain. A brute-force approach is to automatically mesh the cracked domain whenever

* Corresponding author.
E-mail addresses: yongxing.shen@sjtu.edu.cn (Y. Shen), pullding@sjtu.edu.cn (C. Wu), yang.wan@sjtu.edu.cn (Y. Wan).

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the crack is extended. An improvement can be achieved by only re-triangulating the surrounding region of the crack tip, such as the approaches given in [25,26].

In [27], Negri proposed a mesh adaptation method for free discontinuity problems such as Mode III fracture, which does not need to re-mesh. This approach was later adopted by Fraternali [28] for in-plane crack problems.

Proposed by Rangarajan and Lew [29], the method of universal meshes (UM) provides an efficient approach to generate conforming meshes for evolving closed domains with a smooth boundary, using a fixed background mesh, see also [30]. As a result, only a fraction of the vertices, basically those near the evolving boundary of the domain, have to be adjusted: some projected to their respective closest point on the boundary, and some undergoing a relaxation procedure to improve the quality of resulting elements. The applicability of the method rests on the following three requirements:

1. The evolving boundary has to be C^2-continuous, or at least piecewise C^2-continuous;
2. The background mesh consists of only acute triangles;
3. The background mesh is sufficiently refined in the vicinity of the evolving boundary; more precisely, the mesh size does not exceed one fifth of the radius of the curvature of the evolving boundary.

The reader is referred to more recent developments of UM with applications to time-dependent problems with prescribed boundary evolution by Gawlik and Lew [30], to obstacle problems in a low Reynolds number flow by Gawlik et al. [31], to a surface with no boundary in three dimensions by Kabaria and Lew [32], and to a three-dimensional domain by Kabaria [33].

Rangarajan et al. [34] and later Chiaramonte et al. [35] applied the approach to account for a propagating simple crack. Here for convenience of explanation, a distinction of a "positive" side and a "negative" side of the crack is needed, although the choice is immaterial for a simple crack. Then if a triangular element has exactly two vertices on the positive side, it is called a "positive triangle," and the edge formed by said two vertices is called a "positive edge." The essence of UM consists in finding a triangle-free chain of vertices, and then projecting such vertices onto the crack path, at the same time relaxing neighboring vertices. This triangle-free chain is mostly made of positive nodes but adjustment is allowed to ensure that no three vertices that form a triangular element are all chosen in the list.

In this work, we generalize the method presented in [34] to a branched crack, i.e., a crack with one or multiple junctions. A junction can be a triple one or a quadruple one, or one with more than four branches. The essence of the proposed method is a robust manner to determine the "positive side" for each branch, from which most of the vertices to be projected to the corresponding branch will be selected. The determination of these positive sides is based on the desire to alleviate mesh distortion, and is no longer arbitrary as opposed to the case of a simple crack.

The main idea of the proposed method is as follows. Take the case of a triple junction as an example. For the moment, assume that there is a vertex coincident with the junction. Then if this vertex has n neighboring vertices, i.e., shared by n edges, n usually varying between 4 and 8, then the number of ways to associate each of the three crack branches with any of the n edges does not exceed $P_n^3 = n(n-1)(n-2)$. Note that this counting already covers pathological cases with two branches forming a small angle. We then define an algebraic objective function, with each combination above as the variable, that characterizes the mesh distortion that would occur when UM is applied according to the chosen combination. With this setup, we can select the minimizer as the choice of pairing, and this selection also determines the positive sides for UM to be applied later.

The case of multiple junctions in the same crack does *not* introduce much extra complexity. Only a constraint that the positive side of any simple branch between two junctions should be on the same side has to be enforced in the process of searching these positive sides.

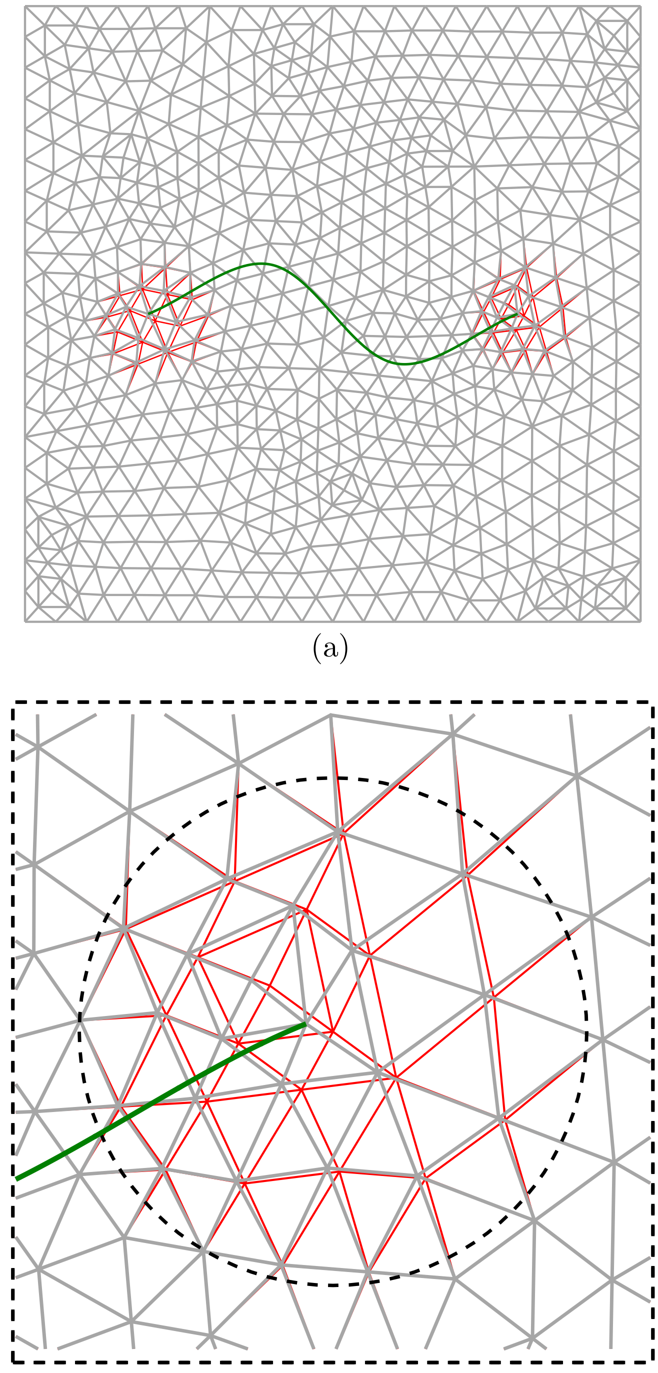

Fig. 1. Illustration of Step 1 for a simple crack: Moving the closest nodes to coincide with the tips and moving the neighboring nodes according to (2). (a) A cracked domain, with an initially non-conforming mesh in red, and the mesh after Step 1 in gray. The crack is shown in green. (b) Magnification of the vicinity of the right crack tip of (a). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The proposed method inherits the above three requirements on the crack path and the mesh for using UM, and also maintains the key advantages of UM, i.e., obtaining the conforming mesh by only perturbing a few vertices of the fixed background mesh.

The structure of the rest of the paper is as follows. In Section 2 we