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Total Lagrangian FEM formulation for nonlinear dynamics of sliding connections in viscoelastic plane structures and mechanisms

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ABSTRACT

In this work we extend a total Lagrangian formulation developed for dynamical nonlinear analysis of elastic plane frames to include structural energy dissipation and sliding connections (as prismatic and cylindrical joints). The joints move in a path that may present an arbitrary roughness profile along its trajectory. Their exact kinematical restraints are introduced in the total mechanical energy of the system by the method of Lagrange multipliers. The structural energy dissipation is considered by a modified Kelvin-Voigt rheological model, for which a pure numerical procedure for the strain rate approximation (finite difference) is proposed. The dynamical equilibrium equation is obtained by a variational principle and the nonlinear solution procedure is done by the Newton-Raphson method. Several examples are presented to demonstrate the efficiency of the proposed formulations.

1. Introduction

The consideration of flexibility in dynamical systems with sliding connections is essential in Engineering practice due to the crescent improvements in the materials employed. The enhancements and specialization of such materials lead to the use of increasingly slenderer and lighter structures and mechanisms for which a rigid body or a linear geometrical analysis is not suitable. Moreover, the mechanical system may also exhibit important dissipative properties which are intrinsic to the material and depart from the purely elastic situation.

Formulations capable to accurately reproduce this complex behaviour of the solid during its overall motion are of great interest. Some applications that include nonlinear dynamics and sliding connections can be cited: satellite antennas, robot arms and cranes, as well as the interaction between vehicle motion and the structural response of bridges.

To properly represent the behaviour of such structures and mechanisms, numerical methods need to correctly describe large displacements and rotations together with the consideration of non-monolithic connections between the parts of a mechanical system, such as, particularly to this study, prismatic and cylindrical joints. It is important to stress that in this work we are worried with viscous dissipation inside the members of structures and mechanisms, this dissipation can be also used when relative orthogonal movement between these parts are present. If a more consistent approach of frictional contact is of interest, readers are invited to consult, for example, the work [1].

Differently from what is proposed in this work, the most disseminated formulations for dynamical analysis of sliding connections in the Finite Element Method (FEM) are based on the updated Lagrangian approach, mainly co-rotational formulations as presented in [2–5]. Additionally, the consideration of kinematic pairs is usually done by means of multibody dynamics formalisms [6–9], which, ultimately, use the background of rigid body mechanics to generate the floating, or shadow, frame approach [10,11] which extends the linear theory to a moving reference frame in a direct manner. The assumption of small deformations in the intermediate frames made by those theories leads to an inexact displacement-strain relation and introduces error accumulation for long term updating processes [12–14]. Also, since the reference system is not the material one, the mass matrix varies according to the frame orientation, requiring the use of special time integration schemes [15–20].

Conversely, in this study we approach the problem of a constrained mechanical system by a total Lagrangian FEM formulation based on the nonlinear solid mechanics theory developed for large deformation analyses, following [21,22]. This formulation is originally extended here to comprise sliding connections and dissipation for plane frame and flexible mechanisms dynamical analysis. The Saint-Venant-Kirchhoff constitutive model is employed to define the solid elastic strain energy using the Green-Lagrange strain and the second Piola-

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Kirchhoff stress tensor. Furthermore, since in this technique velocity and acceleration are Lagrangian variables, which depends upon a fixed reference frame, the mass matrix is constant for the entire analysis and the Newmark algorithm is applicable as enlightened by [13,23–27]. Although our formulation is total Lagrangian, as the majority of approaches in literature are updated Lagrangian or co-rotational, we redirect readers to the important works [28,29] regarding the so called energy conserving and energy dissipating time integrators.

Since the adopted finite element uses positions as the main degrees of freedom, instead of displacements, the kinematical constraints imposed by the joints are introduced very simply in the total energy of the system by means of Lagrange multipliers. We also introduce an arbitrary roughness profile in the trajectory in a straightforward manner allowing diverse modelling options. It is important to mention that exists other approaches to introduce kinematical constrains such as penalty-based techniques [30–32] and master-slave elements [2,33–35]. However, none of the consulted literature includes arbitrary roughness profile in its formulation, which is usually considered by means of equivalent forces or simplified mass/spring/damper systems [36–39].

Internal material damping effects can be introduced in the system as an intrinsic property of the solid, in contrast to velocity-dependent environmental dissipations such as air drag, to simulate, for example, shock absorbers in suspension systems. For this reason, we develop a viscoelastic model for the plane frame which is a modified version of the Kelvin-Voigt rheological model. The usual treatment of linear and nonlinear viscoelastic formulations is done by decaying functions for the material properties leading to frequency domain analysis and convolution integrals as explained by [40–46]. It is important to note that the consulted works that deals with large strains and viscoelasticity adopts updated Lagrangian approaches.

The proposed approach to solve the strain rate is total Lagrangian and purely numerical, i.e., does not use convolutional processes to solve viscosity. It is inspired in the simple one dimensional vibration absorber presented by [47] in which the Green strain measure is used for nonlinear trusses analysis and in the works [48–50] that uses purely numerical integration of viscoelastic models for linear applications, in which a relation of creep and relaxation functions and the viscoelastic constants is proposed.

The organization of this work is as follows. The kinematics of the plane frame element is explained in Section 2. In Section 3 the equations of motion are derived for the unconstrained mechanical system introducing the proposed viscoelastic model. Sections 4 and 5 depict the kinematical constraints imposed by prismatic and cylindrical joints and its consideration in the equation of motion, respectively. Section 6 presents the nonlinear system solution procedure along with the time integration schemes adopted. Section 7 indicates how to calculate the frame internal efforts and Section 8 shows several examples proving the accuracy of the proposed formulation as well as its application to practical engineering problems.

2. Plane frame kinematics

The adopted FEM solution procedure is obtained from the principle of stationary total mechanical energy and, therefore, the strain energy stored in the frame element must be calculated. To calculate the strain energy, it is necessary to define the way one achieves the strain distribution as a function of the body position, limited to a finite number of degrees of freedom. In order to do so, we write the initial and current positions of a frame element as a function of its nodal positions, angles and non-dimensional variables.

2.1. Initial configuration

To build the total Lagrangian procedure we start describing the initial configuration of a frame element. Fig. 1 shows the reference line

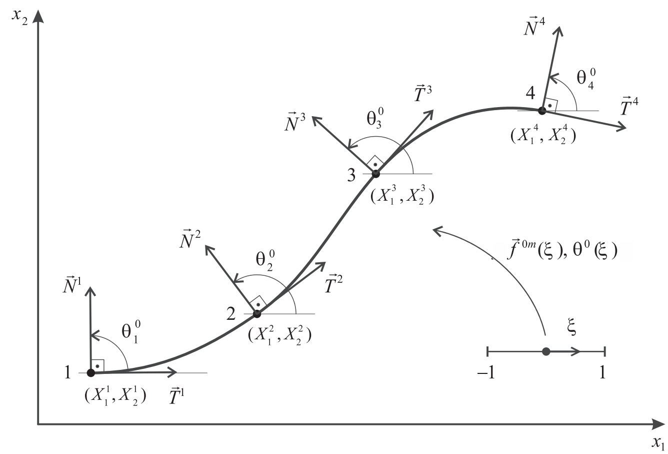

Fig. 1. Reference line parameterization for the initial configuration (cubic finite element approximation).

of the initial configuration, the non-dimensional space from which the reference line mapping is written and the nodes of the finite element.

The mapping of the reference line x_i^m is written by the function f_i^{0m} as:

$$f_i^{0m}(\xi) = x_i^m(\xi) = \phi_\ell(\xi)X_i^\ell \tag{1}$$

where, i is the direction (1 or 2), m indicates reference line and ℓ is the index that represents nodes and shape functions $\phi_\ell(\xi)$ (Lagrange polynomials of any order). Nodal coordinates at the reference line are designated by X_i^ℓ and repeated indices indicate summation.

At the initial configuration, the nodal coordinates are known and the cross sections are considered orthogonal to the reference line. Thus, from Fig. 1, one calculates the tangent vectors at each node k by:

$$T_i^k = x_{i,\xi}(\xi_k) = \phi_{\ell,\xi}(\xi_k)X_i^\ell \tag{2}$$

in which comma indicates derivative.

From Eq. (2), the normal vectors that generate cross sections at the nodes are written as:

$$N_1^k = -T_2^k/\|\vec{T}\| \quad \text{and} \quad N_2^k = T_1^k/\|\vec{T}\| \tag{3}$$

where $\|\vec{T}\|$ is the Euclidean norm of the nodal tangent vector.

To use angles as nodal parameters, one calculates, from Fig. 1:

$$\theta_k^0 = \arctan(N_2^{(k)}/N_1^{(k)}) \tag{4}$$

in which parenthesis indicate no summation.

Knowing the element nodal angles, one can define a cross section at any position along the frame element using the same approximation adopted for coordinates, i.e.:

$$\theta^0(\xi) = \phi_\ell(\xi)\theta_\ell^0 \tag{5}$$

From the reference line initial position and cross sections orientation one defines the frame element initial configuration as depicted in Fig. 2.

One can understand from Fig. 2 that the initial configuration mapping is written using a non-dimensional space (ξ, η) by:

$$x_i(\xi, \eta) = x_i^m(\xi) + g_i^0(\xi, \eta) \tag{6}$$

where the vector $g_i^0(\xi, \eta)$ is a function of $\theta^0(\xi)$, η and the height h_0 of the finite element, given as:

$$g_1^0(\xi, \eta) = \frac{h_0}{2}\eta\cos[\phi_\ell(\xi)\theta_\ell^0] \quad \text{and} \quad g_2^0(\xi, \eta) = \frac{h_0}{2}\eta\sin[\phi_\ell(\xi)\theta_\ell^0] \tag{7}$$

Substituting Eqs. (1) and (7) into Eq. (6) results the complete initial configuration mapping of the frame element, as: