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Mixed dimensional modeling of reinforced structures

Benoît Lé, Grégory Legrain*, Nicolas Moës

GeM, UMR CNRS 6183, École Centrale de Nantes, 1 rue de la Noë 44321, Nantes, France

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ABSTRACT

Using thin reinforcements is a common way to strengthen structures, as in reinforced concrete for example. From a numerical point of view, dealing with these reinforcements is tedious, because of their diameter which is usually small compared to the characteristic dimensions of the structures, therefore requiring very fine meshes to represent them accurately. In this paper, a new approach allowing to mix a volumic and a lineic modeling of the reinforcements is proposed. Fine meshes with a volumic representation of the reinforcements are used in the zones of interest of the structure, whereas coarser meshes associated to lineic elements are used in the rest of the structure in order to decrease the computing times. A methodology to ensure the transition between both modeling is proposed, so that the results in the zones of interest are similar to the results that would be obtained with a full volumic representation of the reinforcements. The efficiency of the method is illustrated on several examples, involving linear elasticity and plasticity.

1. Introduction

Including “almost-1D” reinforcements, that is to say elements with one dimension much greater than the two others, is a common way to strengthen structures. A well-known example is reinforced concrete structures, where steel reinforcements are used in the zones where tensile stresses prevail, to make up for the low tensile strength of concrete. Numerical simulation of such structures is quite challenging from a geometrical point of view. Indeed, the diameter of the reinforcements is usually small compared to the characteristic dimensions of the structures (at least one order of magnitude smaller in the example of reinforced concrete structures), requiring very fine finite element meshes to represent them accurately. As a result, the necessary CPU resources and computation times can become very high.

In order to take into account these reinforcements with reasonable computing costs, different approaches were developed in the literature. The smeared model [1] consists in adding the stiffness of the reinforcements to the volumic elements containing them, introducing orthotropy in the reinforcements direction. This approach is well suited for structures were the reinforcements are perfectly bonded and are arranged in a regular pattern. More recently, David [2] developed a membrane model for regularly spaced reinforcements, based on asymptotic expansions, which is more accurate and allows to take into account loss of bond. In the discrete model, 1D bar elements are added along the edges of the volumic elements. It is more flexible than the smeared approach, since the layout of the reinforcements does not need to be regular anymore, but their paths still need to follow the nodes of the volumic mesh. Finally, the embedded approach [3,4] allows any reinforcements layout, independently of the volumic mesh.

These approaches give good global results, but are not designed to get accurate local results around the reinforcements, as shown in [5] for instance. To sum up, there are two possibilities to model the reinforcements: using a 3D volumic mesh, which would give the most accurate results but is incompatible with industrial studies because of the too large computing costs, or using a smeared or a 1D representation of the reinforcements, decreasing the computing costs but leading to inaccurate local results. The approach proposed in this paper rests upon the idea that, when performing the finite element analysis of any structure, it is often possible to identify zones of interest, that is to say, parts of the structures where accurate results are wanted (because of stress concentrations, or because these are critical parts for which one wants to know how it will deteriorate, etc). Usually, fine meshes are used in such zones, whereas coarser meshes are used in the rest of the structure to decrease computing costs. Now, applying this idea to reinforced structures, we propose in this paper a method that allows to use a volumic representation of the reinforcements in the zone of interest, and a 1D representation in the rest of the structure. The volumic representation in the zone of interest will allow to get results as accurate as possible, especially close to the reinforcements, whereas the 1D representation will allow to use coarser meshes away from the zone of interest, therefore reducing computing times. The transition between both representations will ensure that the results in the fine zone will be close to the results that would have been obtained with a full volumic representation of the reinforcements in the whole structure.

* Corresponding author.
E-mail addresses: benoit.le@ec-nantes.fr (B. Lé), gregory.legrain@ec-nantes.fr (G. Legrain), nicolas.moes@ec-nantes.fr (N. Moës).

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Combining 1D/2D and 3D models in the same computation has already been done, as in [6,7] where coupling 1D beams or 2D plates to 3D bodies was achieved by equating the work at the interface between both representation, resulting in constraint equations between 1D/2D and 3D degrees of freedom. In [8], Nitsche's method [9] is applied to the beam/solid and plate/solid coupling. However, the main difficulty with the existing methods is the need to ensure the compatibility between the beam/plate particular kinematic and the volumic one. In this paper, we will consider structures where the reinforcements have a small enough diameter so that we can assume that they behave like bar element, working in tension/compression. However the proposed approach would still be useful if the bending energy was taken into account [10].

This paper is organized as follow: Sections 2 and 3 introduce the numerical tools involved in the proposed method, as well as their limitations. In Section 4, the solution to combine these tools in order to answer the issues raised above is explained. Some results to illustrate this method are shown in Section 5. Finally conclusions are drawn in Section 6.

2. Volumic model

The most direct approach to model reinforcements would be to mesh them using volumic finite elements. This process may be complicated, considering their small diameter, their number, and their path which may be complicated (intersecting or tangent reinforcements, curved paths, etc). However, the eXtended Finite Element Method (X-FEM) can be used, as done in the work of [11]. This method was first introduced by Moës et al. [12] to deal with crack propagation, with meshes independent of the crack path. Based on the concept of Partition of Unity [13], it relies on the use of enrichment functions to introduce discontinuities into the classical finite elements, as well as level-set functions to locate these discontinuities. The enrichment strategy proposed in [14], dedicated to the analysis of inclusions, is used in this paper for the volumic part of the reinforcements, and will be reminded below.

2.1. The eXtended finite element method (X-FEM)

Consider the bidimensional problem depicted in Fig. 1 a, a circular inclusion made of a material A, in a rectangular plate made of a material B. The plate is meshed with elements whose nodes do not coincide with the material interface. The set I denotes the nodes of the mesh, whereas J is the set of the nodes which need to be enriched. J is defined as the set of nodes belonging to the elements crossed by the material interface (cf. Fig. 1a). The X-FEM approximation consists in finding a displacement solution of the form:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i\in I} N_i(\mathbf{x})\mathbf{u}_i + \sum_{j\in J} N_j(\mathbf{x})F(\mathbf{x})\mathbf{a}_j \quad \mathbf{u}_i,\, \mathbf{a}_j \in \mathbb{R}^2 \tag{1}$$

where $(N_k)_{k=i,j}$ are the classical finite element approximation functions of node k, $\mathbf{u}_i$ are the classical finite element degrees of freedom, $\mathbf{a}_i$ are the enriched degrees of freedom and F is the enrichment function. In this paper we consider that the two materials are perfectly bonded: therefore, $\mathbf{u}^h$ must be continuous over the domain. However, because of the change of material properties, the strain will be discontinuous at the interface. The enrichment function F is thus chosen continuous, but with a discontinuous derivative.

To define F, the position of the interface must be determined. A level-set function ϕ is introduced to locate the interface Γ between material A and B so that:

$$\Gamma = \{\mathbf{x} \in \mathbb{R}^2 \colon \phi(\mathbf{x}) = 0\} \tag{2}$$

$\phi(\mathbf{x})$ is chosen to be positive if $\mathbf{x}$ is outside Γ, negative if $\mathbf{x}$ is inside Γ and equals zero if $\mathbf{x}$ is on Γ. An example of level-set defining the inclusion of Fig. 1a. is given in Fig. 1b. The main example of level-set function is the signed distance function to the interface:

$$\phi(\mathbf{x}) = \pm \min_{\mathbf{x}_\Gamma\in\Gamma} \|\, \mathbf{x} - \mathbf{x}_\Gamma \,\| \tag{3}$$

In the particular case of a cylindrical reinforcement, we use the following level-set function:

$$\phi(\mathbf{x}) = \min_{\mathbf{x}_\Lambda\in\Lambda} \|\, \mathbf{x} - \mathbf{x}_\Lambda \,\| - r \tag{4}$$

where r is the radius of the reinforcement and Λ is its center-line (the 1D curve defining the reinforcement path through its center). From a numerical point of view, ϕ is discretized using the linear finite element shape functions N_i :

$$\phi(\mathbf{x}) = \sum_{i\in I} N_i(\mathbf{x})\phi_i \tag{5}$$

where the ϕ_i are the nodal values of the level-set function. Now that the position of the interface is known, the enrichment function can be defined. We consider the ridge function from [14]:

$$F(\mathbf{x}) = \sum_{i\in I} N_i(\mathbf{x})|\phi_i| - \left|\sum_{i\in I} N_i(\mathbf{x})\phi_i\right| \tag{6}$$

F is shown in 1D on Fig. 2.

Some authors also proposed to treat material interfaces at the integration point level [15]. However, as illustrated in Appendix A, this approach has a lower convergence rate than the X-FEM. On the contrary, for a given mesh density, the X-FEM and conforming FEM lead to similar results without meshing burden, which motivates the use of the X-FEM here.

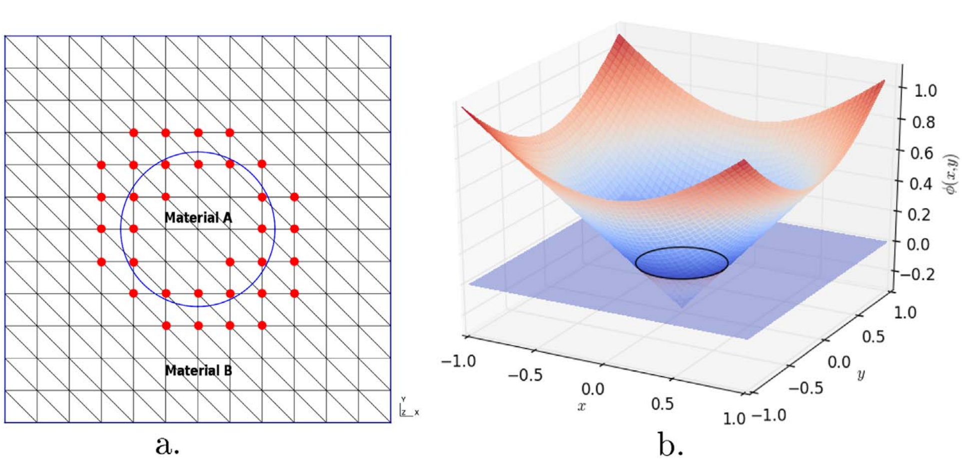

Fig. 1. Representation of a circular inclusion in a square plate, using the X-FEM method. a. Definition of the enriched nodes. b. Localization of the interface using a level-set function.