Finite Elements in Analysis and Design 122 (2016) 1–15

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Hybrid Asynchronous Perfectly Matched Layer for seismic wave propagation in unbounded domains

Michael Brun*, Eliass Zafati, Irini Djeran-Maigre, Florent Prunier

Université de Lyon, INSA-Lyon, LGCIE, 34 rue des Arts, F-69621 Villeurbanne Cedex, France

ARTICLE INFO

Article history:
Received 14 March 2016
Received in revised form
6 July 2016
Accepted 12 July 2016

Keywords:
Perfect Matched Layers (PML)
Elastic wave
Transient analysis
Subdomain coupling
Hybrid Asynchronous PML

ABSTRACT

Perfectly Matched Layer (PML) is recognized as a very effective tool for modeling unbounded domains. Nonetheless, the computation time required by the PML may be large, especially when an explicit time integration scheme is adopted for dealing with the wave propagation problem both in the domain of interest and in the PML medium. In this paper, it is proposed to investigate subdomain strategies enabling the appropriate time integration scheme in the PML with its own time step to be chosen, independently of the choice of the time scheme in the domain of interest. We focus on explicit time integrator in the physical subdomain (Central Difference scheme) associated with a fine time step satisfying the CFL stability criterion. The PML formulation proposed by Basu and Chopra (2004) [1] for 2D transient dynamics, has been coupled with the interior physical subdomain using the dual Schur approach proposed by Gravouil and Combescure (2001) [2]. Hybrid (implicit time integrator for the PML) asynchronous (multi time steps) PMLs have been derived. Their very good accuracy has been shown by considering the following numerical examples: Lamb's test, loaded rigid strip footing on an half space and a layered half space.

1. Introduction

The simulation of wave motion in unbounded media requires the introduction of artificial boundaries surrounding the bounded computational domain. Several techniques have been developed to reproduce an unbounded domain for elastic wave propagation: the infinite elements (Bettess [3], Su and Wang [4]), appropriate absorbing boundary conditions (Enquist and Majda [5]), absorbing layer methods such as the Rayleigh damping layers (Semblat et al. [6], Rajagopal et al. [7]) or the Perfectly Matched Layers (PML) (Chew and Liu [8]). The perfectly matched layers (PML) is an absorbing layer method which surrounds the computational domain with an uniform thickness layer. The PMLs are characterized by their capabilities of providing the same attenuation for all frequencies and all angles of incidence without any reflection from the interface.

The PML was originally developed for the electromagnetic waves by Bérenger [9] using a field-splitting formulation and became one of the most widely used methods in the simulation of wave propagation problems in unbounded media. The technique was then adapted to the elastodynamic equations. Hastings et al. [10] extended the PML from electromagnetics to elastodynamics using a formulation in terms of displacement potentials implemented in the finite difference framework. Using also the finite difference method, Chew and Liu [8] introduced a new split-field formulation for isotropic media, based on the velocity and stress fields. Later on, Collino and Tsoga [11] proposed a finite difference split-field formulation similar to Chew et al. [8], applied to anisotropic media. In [12], Wang et al. developed a new PML formulation, called C-PML based on unsplit-field formulation, using convolution features adapted to the finite difference method. Next, Matzen [13] extended the C-PML approach to the finite element method. In this work, we focus on the unsplit-field formulation in the framework of the finite element method developed by Basu and Chopra for applications involving 2D media [14,1]. More recently, this formulation was extended by Basu to 3D media in the framework of explicit computations [15] and implemented in the FE code LS-DYNA [16]. From the frequency-domain equations of Basu and Chopra, Kucukcoban and Kallivokas derived an unsplit mixed approach of the PML, by retaining the displacement and stress fields as unknowns in the time domain [17]. Next, in order to couple their PML to a displacement-only field formulation in

* Corresponding author.
E-mail addresses: michael.brun@insa-lyon.fr (M. Brun), eliass.zafati@insa-lyon.fr (E. Zafati), irini.djeran-maigre@insa-lyon.fr (I. Djeran-Maigre), florent.prunier@insa-lyon.fr (F. Prunier).

http://dx.doi.org/10.1016/j.finel.2016.07.006
0168-874X/

the physical interior domain, the authors extended their PML to a mixed hybrid approach [18,19].

Following the same idea, it is of great interest to adopt an efficient explicit time integration with a fine time step for the wave propagation into the soil medium without constraining the choice of the time integrators and time step size for the other partitions of the complex soil–structure interaction problem. Indeed, under an earthquake excitation, the problem is multi physics by nature with phenomena occurring at very different space and time scales into the soil and solid media. In this paper, the proposed approach enables the unphysical PML medium to be integrated in time independently of the physical domain. The main benefit is a higher versatility and numerical efficiency of the PML which can be implemented using a more appropriate time integrator associated with a larger time step than the one employed in the soil medium imposed by the CFL condition for ensuring the algorithm stability [20]. The proposed PMLs can be viewed as a hybrid and asynchronous PML version because different time integrators can be adopted as in the work of Kucukcoban and Kallivokas [18,19], with the adding desirable properties of dealing with different time scales in the same dynamic simulation. Moreover, the proposed approach enables the number of unknowns to be reduced in comparison to the previous mixed displacement-stress formulation. More precisely, only displacement quantities are solved in time, at the expense of requiring more storage and calculations involving strain and stress integrals in the PML [1].

The subdomain method proposed by Gravouil and Combescure [21] provides the suitable properties for coupling an explicit time integrator for the subdomain of interest with Newmark implicit time integrators for the other partitions, including the PMLs. The method follows a dual Schur approach by ensuring the velocity continuity at the interface through the use of Lagrange multipliers. The velocity continuity is considered at the fine time scale, that is associated with the time step satisfying the CFL condition. The method is proved to be stable for any Newmark (explicit and implicit) time integrators [22] using the so-called energy method (Hughes, [20]). It leads to the first order of accuracy when coupling second order accurate time integration schemes due to a slight spurious dissipation at the interface as soon as different time steps are adopted. When adopting the same time step, the second order of accuracy is achieved [23]. The GC method was adopted in previous works in order to design implicit, multi directional, multi time step absorbing layers, based on increasing Rayleigh damping ratios in the thickness of the absorbing layers (Zafati et al. [24,25]). Recently, Brun et al. [26] proposed a general framework to derive a family of coupling algorithms from the energy method, initially employed for ODE (Ordinary Differential Equation) and generalized to DAE (Differential Algebraical Equation) after the introduction of the Lagrange multipliers. The coupling algorithms are built by ensuring the zero value at the large time scale of the interface pseudo-energy involved in the generalized energy method. The derived coupling algorithms can be considered as Hybrid Asynchronous Time Integrator (HATI) enabling to couple any Newmark and α schemes, while maintaining the stability and the second order of accuracy of the coupled time integrators [27].

In this paper, the GC method is considered because only Newmark time integrators are investigated. The unsplit field formulation proposed by Basu and Chopra [14,1,28] is adopted and resumed in the first section. The second section is devoted to the coupling algorithm, allowing the use of hybrid multi time step PMLs in explicit computations. The last section concerns numerical examples including Lamb's test and loaded rigid strips lying on the surface of homogeneous and layered soils. In this last two examples, the interest of the proposed approach is highlighted by dealing with three subdomains with different time integrators associated with their own time step: explicit soil subdomain surrounded by implicit multi time step PMLs, and coupled with implicit solid subdomain. Efficiency of the proposed approach is assessed by comparing time histories of displacements and energies as well as L^2 error norms between the numerical results and the reference results obtained by a monolithic full explicit analysis using an extended mesh.

2. Perfectly matched layer

2.1. Strong form of the PML in frequency domain

The PML model used in this work has been developed by Basu and Chopra [1,14]. It is built using the classical elastodynamic equations by introducing the complex-valued stretching functions λ_i. The main idea is to replace the real coordinates x_i with the complex ones $x_i \rightarrow \tilde{x}_i : \mathbb{R} \rightarrow \mathbb{C}$. The complex coordinates are defined by:

$$\frac{\partial \tilde{x}_i}{\partial x_i} = \lambda_i(x_i) = 1 + f_i^e(x_i) - i\frac{f_i^p(x_i)}{bk_s} \tag{1}$$

where b denotes the characteristic length of the physical problem, $k_s = \frac{\omega}{c_s}$ is the wavenumber and c_s is the S-wave velocity. The real-valued positive functions f_i^e and f_i^p vanish at the interface between the PML and the physical domain so that the unphysical PML perfectly matches the physical domain. The damping function f_i^p serves to attenuate the propagating waves in the x_i direction, whereas the damping function f_i^e attenuates the evanescent waves. In Eq. (1), the dependence of the complex term on the factor $i\omega$ allows for an easy application of the inverse Fourier transform when expressing the PML in the time domain, resulting in a PML formulation independent on the frequency. In other words, all the frequencies are damped out in the same way.

The PML formulation is obtained by modifying the governing equations defined in the frequency domain. The classical strong form of the equation of motion for a homogeneous isotropic medium under the plane strain assumption is written by substituting x_i by $\tilde{x}_i$ as follows:

$$\begin{cases} \sum_j \frac{1}{\lambda_j(x_j)} \frac{\partial \sigma_{ij}}{\partial x_j} = -\omega^2 \rho u_j \\ \sigma_{ij} = \sum_{k,l} C_{ijkl} \varepsilon_{ij} \\ \varepsilon_{ij} = \frac{1}{2}\left[\frac{1}{\lambda_j(x_j)} \frac{\partial u_i}{\partial x_j} + \frac{1}{\lambda_i(x_i)} \frac{\partial u_j}{\partial x_i} \right] \end{cases} \tag{2}$$

where C_{ijkl} are the components of the elastic constitutive tensor.

2.2. Strong form of the PML in time domain

Before writing the governing equations of the PML in time domain, we introduce the following notations for the PML region: Ω_{PML} is the region of the PML, bounded by the $\Gamma_{PML} = \Gamma_{PML}^D + \Gamma_{PML}^N$, where $\Gamma_{PML}^D \cap \Gamma_{PML}^N = \varnothing$, defining decomposition of the boundary between Dirichlet and Neumann conditions. In addition, $\underline{g}_N$ denotes the prescribed tractions on ${\Gamma_{PML}}^N$ and $J = [0, T]$ is the time interval of interest. Thanks to the introduction of the stretching functions expressed in Eq. (1), the inverse Fourier transform can