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Identification in transient dynamics using a geometry-based cost function in Finite Element Model Updating method

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ABSTRACT

In this article, we introduce a new numerical method for identifying mechanical parameters of hyperelastic materials in a dynamic framework. Using a Finite Element Model Updating (FEMU) procedure, we propose a new cost function family. The goal is to avoid the use of a random speckle in association with a Digital Image Correlation (DIC) tool that are both needed when a cost function based on full displacement fields is used. The experimental data consist in a set of images. The developed method then uses a simple segmentation of these images without requiring any DIC procedure and associated random speckle technique. This advantage is made possible through the use of a new cost function based on geometry quantities. Examples based on synthetic data illustrate the performance of the proposed method on transient dynamics problems where the flow of information can be very important.

1. Introduction

In a design study, engineers need information on the mechanical characteristics of the used material. To that end, standardized tests are performed on a sample of the considered material. In this article, we are interested in identification methods based on the use of contactless measurements. In that context, a very popular technique is Digital Image Correlation method (DIC) [22] that allows us to obtain the displacement fields: applying a random speckle on the specimen and the different images taken during the test are compared to lead to a full displacement field.

Starting from the displacement fields, different methods have been developed in mechanical engineering to identify material parameters. A review is presented in [2]. Among the classical methods, the Constitutive Equation Gap Method (CEGM) [10,5], the Virtual Field Method (VFM) [11,3,12], the Equilibrium Gap Method (EGM) [6], the Reciprocity Gap Method (RGM) [13] can be mentioned. A very large amount of works are also based on the Finite Element Method Updating (FEMU) principle [14,7] applied in the specific field of dynamics [16]. The method used in this work relies on this FEMU principle: the results of a finite element model are compared to those extracted from images taken during the test.

Major problems are highlighted when random speckle in combination with DIC is used. The first one comes from the realization of the random texture itself whose quality largely depends on the speckle resolution and the resulting contrast. Interesting works are based on goal-oriented filtering technique in which data are combined into new output fields which are strongly correlated with specific quantities of interest [15]. The second problem comes from the DIC and the calculation cost it generates, especially when the number of images to be processed increases sharply. The intrinsic errors related to the DIC can also lead to difficulties, see [1,19,4].

We propose in this article to use geometric quantities (area and second moment of area) as base of the cost function for updating the finite element model. The proposed work thus consists in studying a variant of the FEMU for identify simple behaviors. Tracking random speckle is replaced by a simple detection of the specimen shape in the scene. DIC is therefore replaced by a simple segmentation of the picture easier to implement and much less CPU and memory consuming.

In this work, all data are synthetic. This makes it possible to check the accuracy of the developed method. To its quality, the method is compared to a classical displacement based cost function. We show the developed method is an interesting proposal for simple behaviors.

The paper is organized as follows: in Section 2, we the problem is defined. Section 3 is dedicated to the presentation of the proposed FEMU based identification method. Sections 4–6 present the

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results obtained on numerical tests, and carefully study the effect of various acquisition parameters that could potentially modify the precision.

2. Problem definition

Let us consider $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_n)$ the set of parameters introduced in the constitutive relation of the material defined by:

$$\boldsymbol{S}=\frac{\partial W(\boldsymbol{E};\boldsymbol{\theta})}{\partial \boldsymbol{E}} \tag{1}$$

where $\boldsymbol{S}$ is the second Piola–Kirchhoff stress tensor, $\boldsymbol{E}$ is the Green–Lagrange strain tensor. W is the strain energy density function and depends of material parameters $\boldsymbol{\theta}$.

The mechanical problem is defined considering large deformations in dynamic framework and possible contacts. Furthermore, the material studied is supposed to be isotropic, homogeneous and elastic. Thus, we define the set of equations used in the proposed method of identification.

2.1. Equations of the problem

As shown in Fig. 1, the specimen may be represented as a closed subspace of $\mathcal{R}^2$, denoted Ω, and its boundary $\partial\Omega$ on which one defines the partition:

$$\begin{aligned}&\partial\Omega=\Gamma_u\cup\Gamma_\sigma\cup\Gamma_c,\\&\varnothing=\Gamma_u\cap\Gamma_\sigma=\Gamma_\sigma\cap\Gamma_c=\Gamma_u\cup\Gamma_c,\end{aligned} \tag{2}$$

where the displacements are imposed on Γ_u, the external forces are imposed on Γ_σ and Γ_c is a potential area of contact.

The study is made on the time interval $\tau=[0,t_f]$.

In addition of the constitutive law (1) and within the total Lagrangian formulation, the physical problem is modeled by a set of relationships involving spatio-temporal quantities defined on $\Omega\times\tau$:

- *local equilibrium equations:*

$$\boldsymbol{DivP}+\boldsymbol{b}=\rho\ddot{\boldsymbol{u}} \quad \text{in} \quad \Omega\times\tau \tag{3}$$

  where $\mathbf{P}$ is the first Piola–Kirchhoff stress tensor, $\boldsymbol{b}$ represents the body forces, ρ is the density of the material and $\ddot{\boldsymbol{u}}$ is the acceleration vector.
- *Neumann boundary conditions:*

$$\boldsymbol{P}.\boldsymbol{N}_0=\boldsymbol{T_d} \quad \text{on} \quad \Gamma_\sigma\times\tau \tag{4}$$

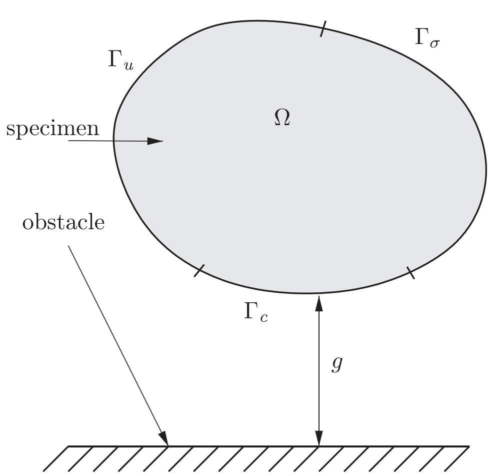

Fig. 1. Continuous problem.

  where T_d is the traction imposed to small surface dS and $\boldsymbol{N_0}$ its normal vector.
- *Dirichlet boundary conditions:*

$$\boldsymbol{u}=\boldsymbol{u_d} \quad \text{on} \quad \Gamma_u\times\tau \tag{5}$$

- *Unilateral contact conditions:*

$$g\geq 0 \quad r_n\geq 0 \quad \text{and} \quad g\cdot r_n=0 \quad \text{on} \quad \Gamma_c \tag{6}$$

where g is the gap between the solid and the obstacle and r_n is the contact reaction force acting along the obstacle outward normal. In this study, the contact is considered without friction.

2.2. Constitutive relations

In this work we use two models of hyper-elastic behavior of which we briefly present the main elements.

- *Saint-Venant Kirchhoff strain energy density function*

In this first case, we identify 2 parameters, $\boldsymbol{\theta}=(E,\nu)$, respectively the Young modulus and Poisson's ratio of the considered material. The strain energy density function, $W(\boldsymbol{E};(E,\nu))$, is defined as a function of $\boldsymbol{E}$ and $\boldsymbol{\theta}=(E,\nu)$ are parameters:

$$W(\boldsymbol{E};(E,\nu))=\frac{E\nu}{2(1+\nu)(1-2\nu)}(\mathrm{tr}(\boldsymbol{E}))^2+\frac{E}{2(1+\nu)}\mathrm{tr}\left(\boldsymbol{E}^2\right) \tag{7}$$

from which we obtain the constitutive relation linking $\boldsymbol{S}$ and $\boldsymbol{E}$:

$$\boldsymbol{S}=\frac{E\nu}{(1+\nu)(1-2\nu)}\mathrm{tr}(\boldsymbol{E}).\boldsymbol{Id}+\frac{E}{(1+\nu)}\boldsymbol{E} \tag{8}$$

where $\boldsymbol{Id}$ is the identity matrix.

- *Blatz–Ko strain energy density function*

In this second case, one parameter is identified, namely the shear modulus, $\boldsymbol{\theta}=(G)$. For practical reason, the elastic deformation potential of this second case refers to the invariants of right Cauchy–Green deformation tensor. The strain energy density function, $W(\boldsymbol{E};(E,\nu))$ is defined by as a function of $\boldsymbol{E}$ and $\boldsymbol{\theta}=(G)$ are parameters:

$$w(\boldsymbol{C};G)=\frac{G}{2}\left[\frac{C_1}{C_2}+2\sqrt{C_3}-5\right]=W(\boldsymbol{E};G) \tag{9}$$

where

$$C_1=\mathrm{tr}(\boldsymbol{C});\quad C_2=\frac{1}{2}\left(C_1^2-\mathrm{tr}\left(\boldsymbol{C}^2\right)\right);\quad C_3=\det\boldsymbol{C} \tag{10}$$

Then:

$$\boldsymbol{S}=\frac{\partial W(\boldsymbol{E};G)}{\partial \boldsymbol{E}}=2\frac{\partial w(\boldsymbol{C};G)}{\partial \boldsymbol{C}} \tag{11}$$

and we classically obtain the constitutive relation:

$$\boldsymbol{S}(\mathbf{E})=G\left\{J(2\boldsymbol{E}+\boldsymbol{Id})^{-1}-(2\boldsymbol{E}+\boldsymbol{Id})^{-2}\right\} \tag{12}$$

where

$$J=\sqrt{\det(2\boldsymbol{E}+\boldsymbol{Id})} \tag{13}$$