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Immersed boundary modal analysis and forced vibration simulation using step boundary method

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ABSTRACT

Analysis using Cartesian background mesh with the geometry embedded or immersed in the mesh is gaining popularity. The primary advantage of this approach is that a traditional mesh, which conforms to the geometry, is not needed. Instead a background mesh that is independent of the geometry and has regular shaped undistorted elements is used because it is easy to generate automatically. Many methods for imposing Dirichlet boundary conditions on immerse boundaries have been studied. In this work step boundary method, where the trial and test functions are weighted using approximate step functions, has been used for imposing Dirichlet boundary conditions on boundaries that do not have nodes on them. This method has been shown to be effective for static problems in the past but has not been studied for dynamics. Step boundary method is extended to modal analysis and modal superposition as well as problems involving base excitation where the Dirichlet boundary conditions are functions of time. Several test examples are used to verify and validate the method.

1. Introduction

Many modifications to the finite element method (FEM) have been proposed for avoiding mesh generation or reducing the difficulties associated with it. A vast amount of literature exits on meshless methods [1] where nodes are not connected into elements for the purpose of interpolation or approximation of the field variables. On the other hand, mesh independent approaches, such as X-FEM [2], use a background mesh that does not conform to the geometry. In XFEM, the geometry (curve or surface) representing a crack is embedded in the mesh and can go through elements. More generally, it is desirable to have the entire geometry defined independent of the mesh [3], using equations of the boundaries as defined in solid models created in computer aided design (CAD) software. The boundaries are surfaces and curves immersed in the background mesh and may not have nodes on them. Imposing Dirichlet (or essential) boundary conditions on the embedded geometry is a key challenge. Popular approaches for imposing Dirichlet boundary conditions on boundaries without nodes include the penalty method, Lagrange Multiplier methods and Nitsche's method. Variations of the Lagrange multiplier approach have been most widely studied in this context including for constraints on embedded interfaces. To avoid instability when using the Lagrange multiplier approach, it is necessary that the field variables and Lagrange multiplier fields satisfy the inf-sup condition [4] for which several stabilization strategies have been developed [5–7]. Nitsche's method has been used for weakly enforcing constraints on embedded interface problems where stabilization parameters are calculated to ensure coercivity [8–10]. A related method was developed by Baiges et al. [11] for imposing Dirichlet boundary conditions where the stability parameter is independent of the numerical approximation.

Several methods that fall into the category of mesh independent analysis using a Cartesian mesh with embedded geometry have been developed and are known by different names such as embedded/immersed domain method [12], fictitious domain method [13] and immersed boundary method [14,15]. Finite Cell Method (FCM) [16–18], uses the fictitious domain approach with high order basis functions to represent the solution while the geometry is embedded in this mesh. In the FCM approach, a stiff strip of material along the boundary has been used to impose boundary conditions and more recently Nitsche's method has been used to weakly impose boundary conditions on embedded boundaries [19] where a careful choice of stabilization parameters yields good convergence. Immersed b-spline (i-spline) finite element method [20] uses modified b-splines for nodes near the boundary so that they locally interpolate the test and trial functions at the boundary and therefore the Dirichlet boundary conditions can be applied simply by specifying nodal values. The resulting basis functions are rational functions and, for integration purposes, local elements that fit the boundary are used. In the present work we study an implicit boundary method for applying Dirichlet boundary conditions on immersed boundaries. Kantorovich and Krylov

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[25] originally proposed using implicit equations of the boundary curves and surfaces to impose boundary conditions by constructing trial solutions of the form $u(\mathbf{x}) = u^h(x)\Phi(\mathbf{x}) + a(\mathbf{x})$, where $\mathbf{x} \in \Omega$, $\Phi: \Omega \to \mathbf{R}$. Here $\Phi(\mathbf{x}) = 0$ is the implicit equation of the boundary, $u^h(x)$ is a piece-wise interpolation / approximation, $a(\mathbf{x})$ is a function whose values match the specified boundary conditions at the boundaries and Ω is the volume of the entire domain. This solution automatically satisfies the Dirichlet boundary condition $u = a$ because at the boundary $\Phi(\mathbf{x}) = 0$. For arbitrary geometry, it is difficult to construct implicit equations and therefore R-functions and signed distance functions [26–30] have been suggested as ways to construct the required characteristic functions $\Phi(\mathbf{x})$.

To avoid poor convergence and difficulties in quadrature due to highly nonlinear characteristic functions, we have used a modified form of the implicit boundary method that uses step functions as the characteristic function. In the present work, we refer to this variation of the implicit boundary method as the 'step boundary method'. It uses approximate step functions to represent Dirichlet boundaries and has been shown to be effective at imposing Dirichlet boundary conditions for many static problems [21–24] when a Cartesian background mesh is used for the analysis. Approximate step functions have a unit value inside the domain and transitions to zero over a small distance at the Dirichlet boundaries. Therefore, only the boundary elements containing Dirichlet boundaries are affected by this characteristic function. A background mesh with uniform, undistorted (regular-shaped) elements is used to construct the piece-wise approximation $u^h(x)$. Therefore quadrature inaccuracies arising due to distorted elements in conforming meshes can be avoided. The implicit boundary method has also been applied when approximation schemes that do not have Kronecker's delta properties, such as B-spline approximations, are used to build trial and test functions [22,23]. B-spline approximations have been used in traditional finite element method [25–33] as well as with isogeometric approach [34–37]. In the isogeometric approach, both the solution and the geometry are approximated using the same basis function. In addition to B-Spline basis functions, NURBS and T-splines have been used as basis function to obtain higher order approximations, h- and p- refinement as well as k-refinement wherein the degree of continuity of the solution is raised to improve the solution quality. Several methods for imposing Dirichlet boundary conditions for these basis functions have been proposed including the use of multiple knots at boundary nodes and utilizing constraint equations. The step boundary method has not been used for isogeometric method but it is a general approach for imposing boundary conditions that can be used for isogeometric method and traditional FEM as well. However, its real benefit is that it can be used with a mesh that does not have nodes on the boundaries of the analysis domain. It requires only the equation of the boundary to impose the boundary conditions. Therefore, it is ideally suited for the immersed boundary approach where the geometry is embedded in a uniform background mesh. The geometry of the structure is assumed to be available as equations, typically from a solid model created in CAD software.

In this work, we extend the step boundary method to structural dynamics problems solved using modal analysis and modal superposition. While this method has been found to be effective for static problems, it has never been tested for vibration problems to study the accuracy with which natural frequencies can be computed. In addition, the method is studied here for forced vibration problems and base excitation problems where the Dirichlet boundary conditions are functions of time. In Section 2, we summarize how geometry is represented using parametric equations and how it is used in the construction of trial and test functions that satisfy the Dirichlet boundary conditions. The step boundary method is extended to dynamics in Section 3, where a modified weak form is derived using trial and test functions described in Section 2. The load vector corresponding to base excitation is derived in Section 4, where both primary base excitation and base excitation due to multiple base motions are discussed. The formulations is validated and studied using several benchmark examples in Section 5.

2. Geometry and trial function representation

In CAD models [38,39], the geometry of curves and surfaces are typically represented using parametric equations of the form $\mathbf{\Gamma}_i(\upsilon)$: $\mathbf{C} \to \mathbf{E}^3$ for curves and $\mathbf{\Gamma}_i(\xi, \eta)$: $\mathbf{A} \to \mathbf{E}^3$ for surfaces, where $\mathbf{\Gamma}_i$ is the position vector of points on the curve or surface, $\mathbf{C}$ is the curve that represents domain in parametric space for curves and $\mathbf{A}$ is an area in the parametric space representing the domain for surfaces. Geometry of solids is defined using Boundary Representation (B-Rep) models. For 2D analysis as well as plate or shell analysis, the geometry can be represented as a face or a collection of faces, often referred to as 'shell' in geometric modeling literature. A face is modeled as a bounded parametric surface whose boundaries are defined using loops. The loops are defined using a collection of oriented edges that are linked together to bound a closed region. Strict conventions are used to facilitate the classification of points on the surface as being inside or outside the face. For example, the direction of the oriented-edges and the associated loops are defined such that as one traverses along the loop in its predefined direction, the inside of the face is to the left. An oriented-edge is in turn defined by associating it to an edge and defining its direction as being either the same or the opposite to that of the edge. The geometry of an edge is a parametric curve and it is bounded by vertices. The direction of these edges is defined as the direction in which one would move along the curve as its parameter υ is increased. The notations used in this paper to describe the geometry of the structures to be analyzed are briefly discussed here.

Let the parametric surface representing a face be $\mathbf{\Gamma}_i(\xi, \eta)$: $\mathbf{A} \to \mathbf{E}^3$ and its boundaries be defined using a set of oriented-edges $\mathbf{\Gamma}_{ij}(\upsilon)$: $\mathbf{C} \to \mathbf{E}^3$, $j = 1..\, n_b$, where, n_b is the number of boundaries and the domain of the j^{th} boundary curve is $[\upsilon_{0j}, \upsilon_{1j}]$. These boundary curves together define a closed region on the surface that may consist of multiple closed loops defining an external and any internal boundaries.

Fig. 1 shows two faces representing a shell where each face is bounded by four oriented-edges. The vertices of the bounding curves are connected such that the end point of an oriented edge in the loop is the starting point of the subsequent oriented edge within the same loop. For 2D and shell-like structures, any point in the volume of the structure can be denoted as

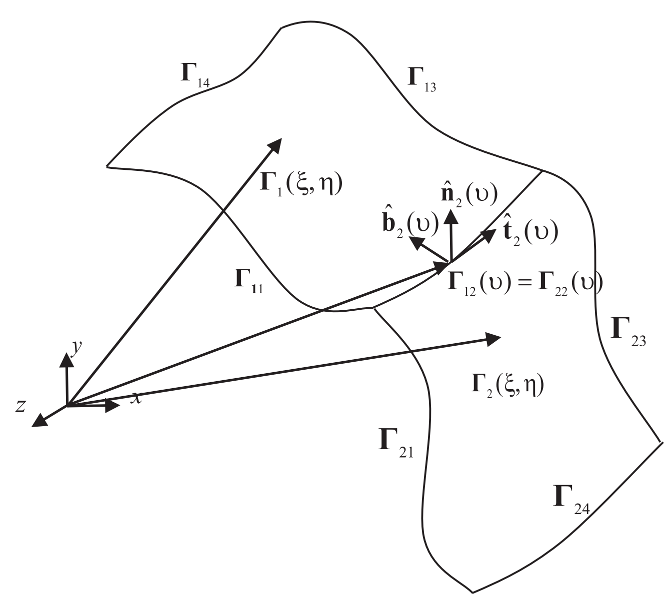

Fig. 1. Faces representing a plane, shell or boundary of a structure.