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A path-following method for elasto-plastic solids and structures based on control of plastic dissipation and plastic work

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A B S T R A C T

A path-following method that is based on controlling plastic dissipation or plastic work in an inelastic solid or structure is presented. It can be effective for highly nonlinear materially and geometrically problems. In particular, it can be applied for elasto-plastic problems where the standard arc-length methods fail, or to avoid artificial and undesirable elastic unloading of a complete solid or structure during the computation. The essential ingredients, the plastic dissipation and the plastic work based constraint equations, are derived by using either explicit or implicit pseudo-time step integration. These constraint equations are valid for geometrically nonlinear small strain elasto-plasticity with hardening. Their implementation in the framework of the path-following method is described. Several numerical examples are presented in order to illustrate very satisfying performance of the derived path-following method. It performed very well for some challenging shell problems.

1. Introduction

The most used path-following method in the nonlinear finite element analysis of solids and structures is probably the Crisfield's cylindrical arc-length, see e.g. [10,11,9]. Other path-following methods are also used, like those originally presented in [28] (this one is implemented in commercial finite element code Abaqus [1]) and [27]. A recent review on rather standard path-following methods, including the above mentioned, can be found in [29]. The standard path-following methods can be successfully applied for solving many geometrically nonlinear problems as well as many types of geometrically and materially nonlinear problems. However, they might fail when computing a particularly demanding nonlinear problem, e.g. a problem related to structural collapse due to material failure. For this kind of problems, several goal-oriented path-following methods were proposed, see e.g. [19,2,20,26,3] and references therein. A specific path-following method is usually designed for a specific class of problems.

The characteristic part of any path-following method is the constraint equation. Recently, Verhoosel et al. [33] presented constraint equations that are controlling energy dissipation in an inelastic material. In [33], several constraint equations were presented, in particular for geometrically linear and geometrically nonlinear elasto-damage, and geometrically linear elasto-plasticity (without hardening).

In this work, we extent the ideas of Verhoosel et al. [33] to geometrically nonlinear elasto-plasticity. In particular, we derive explicit and implicit constraint equations that control plastic dissipation for small strain elasto-plasticity with hardening (see e.g. [21,31] for details on computational elasto-plasticity). The implementation of an explicit constraint equation in the framework of the consistently linearized path-following method, see e.g. [30,17,35], is rather straightforward. Namely, all the ingredients of the explicit constraint equation are already computed in the course of geometrically nonlinear elasto-plastic analysis, e.g. [13], [34]. On the other hand, an implicit constraint equation is much more complex and its implementation is quite demanding.

An application of here presented formulations to embedded discontinuity finite elements, e.g. [14,15,22,23], that are used to model material failures, are presented in [8].

The rest of the paper is organized as follows. In Section, 2 the path-following method framework is presented. In Section 3, several plastic dissipation based constraint equations are derived by using an explicit or implicit integration in the pseudo-time step. Section 4 provides illustrative numerical examples. Conclusions are drawn in Section 5.

2. The framework of the path-following methods

In the nonlinear finite element method for solids and structures, one has to solve the following system of nonlinear equations

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$$\mathbf{R}(\mathbf{u}(t), \lambda(t)) = \mathbf{R}^{\text{int}}(\mathbf{u}(t)) - \mathbf{f}^{ext}(\lambda(t)) = \mathbf{0} \quad (1)$$

where $\mathbf{R}^{\text{int}}$ and $\mathbf{f}^{ext}$ are vectors of internal and external (equivalent) nodal forces (and moments, if they are present in the formulation), respectively, u is vector of unknown nodal displacements (and rotations, if they are present in the formulation, see e.g. [6]), λ is the load factor, and $t \geq 0$ is a monotonically increasing parameter that will be called the pseudo-time. In many practical cases, the system of equations (1) is possible to solve only with an additional constraint equation

$$g(\mathrm{u}(t) - \mathrm{u}(t - \Delta t), \lambda(t) - \lambda(t - \Delta t)) = 0 \quad (2)$$

where Δ represents a small (incremental) change. Solving (1) and (2) simultaneously is called the path-following method or the arc-length method if (2) has a cylindrical or spherical form, see e.g. [10,9]. The solution of (1) and (2) is searched for at the discrete pseudo-time points $0 = t_0, \ t_1, \ \dots, \ t_n, \ t_{n+1}, \ \dots, \ t_{final}$. Let us assume that configuration at t_n is known (the notation $a(t_n) = a_n$ will be used in what follows) and defined by the pair $\{\mathrm{u}(t_n), \lambda(t_n)\} = \{\mathrm{u}_n, \lambda_n\}$. At searching for the next configuration at $t_{n+1} = t_n + \Delta t_n$, we additively decompose u_{n+1} and λ_{n+1} as $\mathrm{u}_{n+1} = \mathrm{u}_n + \Delta\mathrm{u}_n$ and $\lambda_{n+1} = \lambda_n + \Delta\lambda_n$, where $\Delta\mathrm{u}_n$ and $\Delta\lambda_n$ are the increments of the displacement vector and the load vector, respectively. Eqs. (1) and (2) can be rewritten for t_{n+1} as

$$\mathbf{R}_{n+1}(\mathbf{u}_n, \lambda_n; \Delta\mathbf{u}_n, \Delta\lambda_n) = \mathbf{0}, \quad g_{n+1}(\Delta\mathbf{u}_n, \Delta\lambda_n) = 0 \quad (3)$$

where $\Delta\mathrm{u}_n$ and $\Delta\lambda_n$ are the unknowns. The solution of (3) is searched for iteratively by the Newton-Raphson method. At an iteration i, the following linear system has to be solved

$$\begin{bmatrix} \mathbf{K}_{n+1}^i & \mathbf{R}_{n+1,\lambda}^i \\ [g_{n+1,\mathbf{u}}^i]^T & g_{n+1,\lambda}^i \end{bmatrix} \begin{Bmatrix} \Delta\tilde{\mathbf{u}}_n^i \\ \Delta\tilde{\lambda}_n^i \end{Bmatrix} = - \begin{Bmatrix} \mathbf{R}_{n+1}^i \\ g_{n+1}^i \end{Bmatrix} \quad (4)$$

for the iterative pair $\{\Delta\tilde{u}_n^i, \Delta\tilde{\lambda}_n^i\}$, where $(\circ)_{,\lambda}$ and $(\circ)_{,\mathbf{u}}$ denote the derivatives of $(\circ)$ with respect to $\Delta\lambda_n^i$ and $\Delta\mathbf{u}_n^i$, respectively, and $\mathbf{K}_{n+1}^i = \mathbf{R}_{n+1,\mathbf{u}}^i$ is the tangent stiffness matrix. New iterative guess is obtained as $\Delta\mathrm{u}_n^{i+1} = \Delta\mathrm{u}_n^i + \Delta\tilde{\mathrm{u}}_n^i$ and $\Delta\lambda_n^{i+1} = \Delta\lambda_n^i + \Delta\tilde{\lambda}_n^i$. System of Eq. (4) can be effectively solved by the bordering algorithm, see e.g. [35] for details. When the iteration loop ends due to fulfilment of a convergence criterion, the converged incremental values $\Delta\mathrm{u}_n$ and $\Delta\lambda_n$ are obtained. The configuration $\{\mathrm{u}_{n+1}, \lambda_{n+1}\}$ at t_{n+1} becomes known and the search for the solution at the next pseudo-time point can start.

The above framework is valid for any constraint function g_{n+1} in (3). However, the robustness and efficiency of the path-following method depend crucially on the specific form of this function. In what follows, we will elaborate for the case when g_{n+1} controls the incremental structural plastic dissipation when elasto-plastic or rigid-plastic material models are used, see e.g. [10,21,31] for such models.

3. Plastic dissipation constraint equation for geometrically nonlinear elasto-plasticity

In this section, we will present and discuss several possibilities for deriving the constraint equation $g_{n+1} = 0$, see (3), which will control incremental structural plastic dissipation. In particular, we will derive the plastic dissipation constraint equation by two different approaches (called version 1 and version 2) and we will show in Section 3.3 that the final results of those two approaches are equivalent.

3.1. Explicit form of plastic dissipation constraint equation – version 1

The rate of plastic dissipation in an elasto-plastic solid or structure can be defined as $\dot{D} = \dot{P} - \dot{\Psi}$, where $\dot{P}$ is the pseudo-time rate of the applied work, and $\dot{\Psi}$ is the pseudo-time rate of the thermodynamic (i.e. the free energy) potential for plasticity (the dot denotes the derivative with respect to the pseudo-time). For the discretized solid or structure in the framework of the geometrically nonlinear and inelastic finite element method, $\dot{P}$ can be written as

$$\dot{P} = \sum_e \int_{V^e} \mathbf{S}^T \dot{\mathbf{E}} dV = \mathbf{f}^{ext,T} \dot{\mathbf{u}} = \lambda \hat{\mathbf{f}}^{ext,T} \dot{\mathbf{u}} \quad (5)$$

where e denotes a finite element of the mesh, $\mathbf{S}$ and $\mathbf{E}$ are vectors comprising the 2nd Piola-Kirchhoff stresses and the Green-Lagrange strains, respectively, and V^e is the initial volume of the element. It was assumed in (5) that the external forces can be expressed as $\mathrm{f}^{ext} = \lambda \hat{\mathrm{f}}^{ext}$, where $\hat{\mathrm{f}}^{ext}$ is a fixed pattern of nodal forces. The free energy potential of a solid or structure, based on the St. Venant-Kirchhoff elasticity and plasticity with linear isotropic hardening, is $\Psi = U + H$, where the stored energy due to elastic deformations is

$$U = \sum_e \int_{V^e} \frac{1}{2} \mathbf{E}^{\text{el},T} \mathbf{D} \mathbf{E}^{el} dV = \sum_e \int_{V^e} \frac{1}{2} \mathbf{S}^T \mathbf{D}^{-1} \mathbf{S} dV \quad (6)$$

and the stored energy due to the material hardening is

$$H = \sum_e \int_{V^e} \frac{1}{2} K_h \xi_h^2 dV \quad (7)$$

Here, $\mathbf{E}^{el} = \mathbf{E} - \mathbf{E}^p$ is vector of elastic strains, $\mathbf{E}^p$ is vector of plastic strains, D is symmetric constitutive matrix that relates stresses with elastic strains $\mathbf{S} = \mathbf{D}\mathbf{E}^{el}$, K_h is hardening modulus, and ξ_h is strain-like variable that controls linear isotropic hardening. For any other type of hardening, H in (7) has to be changed accordingly. Differentiation of U with respect to the pseudo-time gives

$$\dot{U} = \sum_e \int_{V^e} \dot{\mathbf{E}}^{el,T} \mathbf{C}^{ep} \mathbf{D}^{-1} \mathbf{S} dV = \dot{\mathbf{u}}^T \underset{e}{A} \left[\int_{V^e} \mathbf{B}^T \mathbf{C}^{ep} \mathbf{D}^{-1} \mathbf{S} dV \right] \quad (8)$$

where C^{ep} and B denote the consistent symmetric elasto-plastic tangent modulus and the strain-displacement matrix, respectively, and A is the finite element mesh assembly operator. The following relations were used in (8):: $\dot{\mathrm{S}} = \mathrm{C}^{ep}\dot{\mathrm{E}}$, $\dot{\mathbf{E}} = \mathbf{B}\dot{\mathbf{u}}^e$, and $\dot{\mathbf{u}} = \underset{e}{A} \dot{\mathbf{u}}^e$, where $\mathbf{u}^e$ is vector of element nodal displacements. Differentiation of H with respect to the pseudo-time yields

$$\dot{H} = \sum_e \int_{V^e} K_h \xi_h \dot{\xi}_h dV = \sum_e \int_{V^e} K_h \xi_h \left(\frac{\partial \xi_h}{\partial \mathbf{u}^e} \right)^T \dot{\mathbf{u}}^e dV = \dot{\mathbf{u}}^T \underset{e}{A} \left[\int_{V^e} K_h \xi_h \left(\frac{\partial \xi_h}{\partial \mathbf{u}^e} \right) dV \right] \quad (9)$$

Let us use the forward Euler pseudo-time step in order to express the plastic dissipation at the pseudo-time point t_{n+1}, i.e. $D_{n+1} = D_n + \dot{D}_n \Delta t_n$, where $\Delta t_n = t_{n+1} - t_n$. Let us further define the following constraint equation:

$$g_{n+1} = D_{n+1} - D_n - \tau_n = 0 \quad \Rightarrow \quad g_{n+1} = \dot{D}_n \Delta t_n - \tau_n = 0 \quad (10)$$

where τ_n is a predefined (required) value of plastic dissipation at pseudo-time step $[t_n, t_{n+1}]$. By concluding that $\dot{D}_n = \dot{P}_n - \dot{\Psi}_n = \dot{P}_n - \dot{U}_n - \dot{H}_n$ and by using (5), (8) and (9), the Eq. (10) can be rewritten as

$$g_{n+1} = \dot{D}_n \Delta t_n - \tau_n = \Delta\mathbf{u}_n^T (\lambda_n \hat{\mathbf{f}}^{ext} - \mathbf{f}_n^*) - \tau_n = 0 \quad (11)$$

Here, $\Delta\mathbf{u}_n = \dot{\mathbf{u}}_n \Delta t_n$ is the current iterative guess of incremental displacements (iterative index i is omitted) and

$$\mathbf{f}_n^* = \underset{e}{A} \left[\int_{V^e} \mathbf{B}_n^T \mathbf{C}_n^{ep} \mathbf{D}^{-1} \mathbf{S}_n dV + \int_{V^e} K_h \xi_{h,n} \left(\frac{\partial \xi_h}{\partial \mathbf{u}^e} \right)_n dV \right] \quad (12)$$

It follows from (11) and (12) that the derivatives needed in (4) are $g_{n+1,\lambda} = 0$ and $g_{n+1,\mathbf{u}} = \lambda_n \hat{\mathbf{f}}^{ext} - \mathbf{f}_n^*$, since g_{n+1} is not a function of $\Delta\lambda_n$. Most of the terms of $\mathbf{f}_n^*$ in (12) are needed for the elasto-plastic analysis and can be readily used to compute (11) and its derivatives. An exception is $(\partial\xi_h/\partial\mathbf{u}^e)_n$, which can be obtained by using the elasto-plastic constitutive relations.

In practice, one has to compute $\mathbf{f}_n^*$ after the last iteration at t_n or before the first iteration at t_{n+1} and use it when iterating for config-

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