Journal of Computational Physics 350 (2017) 207–236

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A novel consistent and well-balanced algorithm for simulations of multiphase flows on unstructured grids

Jitendra Kumar Patel, Ganesh Natarajan *

Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Assam 781039, India

ARTICLE INFO

Article history:
Received 21 January 2017
Received in revised form 11 June 2017
Accepted 23 August 2017
Available online xxxx

Keywords:
Volume-of-fluid
Consistent transport
Balanced force algorithm
High density ratios
Hybrid staggered/non-staggered
Multiphase flows

ABSTRACT

We discuss the development and assessment of a robust numerical algorithm for simulating multiphase flows with complex interfaces and high density ratios on arbitrary polygonal meshes. The algorithm combines the volume-of-fluid method with an incremental projection approach for incompressible multiphase flows in a novel hybrid staggered/non-staggered framework. The key principles that characterise the algorithm are the consistent treatment of discrete mass and momentum transport and the similar discretisation of force terms appearing in the momentum equation. The former is achieved by invoking identical schemes for convective transport of volume fraction and momentum in the respective discrete equations while the latter is realised by representing the gravity and surface tension terms as gradients of suitable scalars which are then discretised in identical fashion resulting in a balanced formulation. The hybrid staggered/non-staggered framework employed herein solves for the scalar normal momentum at the cell faces, while the volume fraction is computed at the cell centroids. This is shown to naturally lead to similar terms for pressure and its correction in the momentum and pressure correction equations respectively, which are again treated discretely in a similar manner. We show that spurious currents that corrupt the solution may arise both from an unbalanced formulation where forces (gravity and surface tension) are discretised in dissimilar manner and from an inconsistent approach where different schemes are used to convect the mass and momentum, with the latter prominent in flows which are convection-dominant with high density ratios. Interestingly, the inconsistent approach is shown to perform as well as the consistent approach even for high density ratio flows in some cases while it exhibits anomalous behaviour for other scenarios, even at low density ratios. Using a plethora of test problems of increasing complexity, we conclusively demonstrate that the consistent transport and balanced force treatment results in a numerically stable solution procedure and physically consistent results. The algorithm proposed in this study qualifies as a robust approach to simulate multiphase flows with high density ratios on unstructured meshes and may be realised in existing flow solvers with relative ease.

1. Introduction

The interaction of immiscible fluids with distinct interface(s) can be seen in several engineering and environmental phenomena. Understanding these multi-component flows and their underlying physics necessitates the use of high-resolution

* Corresponding author.
E-mail addresses: j.patel@iitg.ernet.in (J.K. Patel), n.ganesh@iitg.ernet.in (G. Natarajan).

http://dx.doi.org/10.1016/j.jcp.2017.08.047

numerical algorithms for their simulations. Unlike incompressible approaches for single-phase flows, studies in multiphase flows are fraught with challenges that include resolving the interface(s) sharply and simulating realistic and high density and viscosity contrasts successfully. In addition, the presence of interfacial forces due to surface tension and volumetric forces such as those due to gravity and electromagnetic effects need to be suitably accounted for in the numerical discretisation to ensure a stable solution procedure.

Multiphase simulations have been carried out using several different approaches over the past two decades. While a comprehensive and exhaustive review of the various methods may be found in [1], we briefly recall the salient contributions in this field of research. Restricting to a single fluid formalism, it is easy to see that the widely used approaches belong to the class of volume-of-fluid (VoF) or the level-set methods with both approaches having their distinct strengths and weaknesses. The VoF approach, which originated from the work of Hirt and Nichols [2] while being discretely conservative leads to discontinuous property variations whereas the level-set approach pioneered by Osher and Sethian [3] results in a smoother transition of fluid properties at the cost of sacrificing conservation. There have been attempts at developing variants such as the mass-conserving level set method (MCLS) [4] and also at hybridising these approaches such as the CLSVOF method [5] and the VOSET method [6], which have met with considerable success. Simulations of large density multiphase flows have also found significant attention in the framework of lattice Boltzmann methods [7] as well as diffuse interface approaches [8].

An important issue that plagues multiphase flow simulations, particularly those involving interfacial tension, is the generation of spurious currents. Spurious currents in surface-tension dominated flows arise due to the errors in calculation of curvature and the discrete imbalance between surface tension and pressure forces [9]. Curvature errors can be minimised using improved approaches for calculating the interface curvature as detailed in [10–12] but can never be completely eliminated and represent a constant source of non-zero spurious currents. Abadie et al. [13] have shown that spurious currents are related to spurious vorticity production and have investigated the role of advection schemes in their evolution in conjunction with different curvature calculation approaches. It is also discussed in [14] that coupling between surface tension force, advection scheme and momentum transport is crucial. A recent study by Pan et al. [15] have shown that spurious currents in low Capillary number flows can be significantly reduced by employing a moving reference frame for simulations. However, one can totally eliminate errors due to force imbalances by carefully constructing a discretisation that enforces balance between competing forces, leading to a class of approaches referred to as "well-balanced" (or "balanced force") algorithms. The principle behind these class of methods is to use identical discretisation of the pressure gradient and surface tension term that appear in the momentum equation and with a reasonably accurate interface curvature calculation leads to acceptably low spurious currents. The earliest and significant contributions to well-balanced algorithms include the works of Renardy and Renardy [16] and Francois et al. [17]. While the former proposed the parabolic reconstruction of surface tension (PROST) for accurate and balanced implementation of interfacial tension, the latter discusses the need for and development of a balanced algorithm for multiphase flows on uniform structured meshes. Their algorithm however involved interpolations of centroidal quantities to faces and vice-versa and did not account for body forces. Montazeri and Ward [18] devised an improved pressure-velocity coupling based on Newton's divided-differences for multiphase flows that is effective on non-uniform meshes and considers interfacial and body forces. Denner and van Wachem [19] have proposed a pressure-velocity coupling for multiphase flows on unstructured meshes with modifications to the original momentum interpolation method for surface tension and gravity forces as well as grid non-orthogonality. Ghods and Herrmann [20] adopt a different approach to achieve the force balance by employing a least-squares based reconstruction for centroidal gradients, although despite its generic nature this approach has not been extensively investigated on unstructured grids for multiphase flows with large body forces. It must be remarked that while many researchers discuss and test balanced force algorithms for interfacial tension dominated flows, they are equally important in presence of other forces as well, such as body forces like gravity [21,22].

A related pertinent challenge that has received relatively lesser attention is the ability of the flow solver to handle large density and viscosity ratios. This is critical because the density and viscosity ratios are $\mathbf{O}(100)$–$\mathbf{O}(1000)$ for engineering applications and the key idea for a robust and stable numerical algorithm at high density ratios is to ensure a consistent transport of mass and momentum and the discrete level. This was first proposed by Rudman within a VoF framework on staggered grids [23] which is also adopted to level-set method by Desjardins and Moureau [24] and later extended to unstructured collocated meshes by Bussmann [25]. Raessi and Pitsch [26] proposed a density-based momentum flux correction in a level-set framework for tightly coupling mass and momentum, where the density fluxes computed from the level-set are used to define the momentum fluxes. More recently, Ghods and Herrmann [20] proposed a consistent rescaled mass-momentum transport (CRMT) algorithm in a level-set framework for multiphase flows. The fundamental philosophy was to convect mass and momentum using the same upwind scheme, then solve the level set equation using a higher-order accurate scheme and recompute the density using the level set information. This led to conservation errors in both mass and momentum, but in a consistent manner, which allowed a stable simulation of large density ratio flows.

The focus of the present work is to develop and investigate a fully balanced and consistent transport algorithm for multiphase flows on arbitrary polygonal meshes. The work employs a novel staggered/non-staggered framework for solving the governing equations and will be shown to naturally lead to a discretisation that motivates a balanced force approach which accounts for interfacial as well as body forces. Following Ghods and Herrmann [20], we implement a consistent mass-momentum transport in the VoF framework with a second-order accurate bounded scheme that is shown to work