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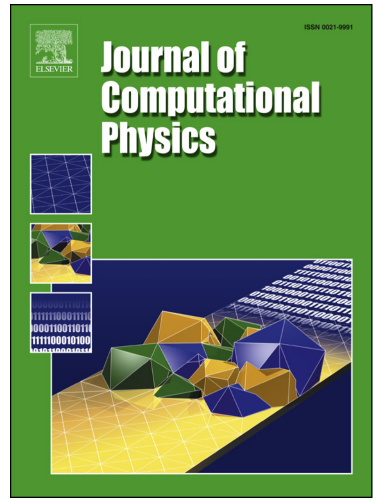

Domain decomposition methods for space fractional partial differential equations

Yingjun Jiang, Xuejun Xu

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Domain Decomposition Methods for Space Fractional Partial Differential Equations*

Yingjun Jiang[†] and Xuejun Xu[‡]

Abstract

A two-level additive Schwarz preconditioner is proposed for solving the algebraic systems resulting from the finite element approximations of space fractional partial differential equations (SFPDEs). It is shown that the condition number of the preconditioned system is bounded by $C(1 + H/\delta)$, where H is the maximum diameter of subdomains and δ is the overlap size among the subdomains. Numerical results are given to support our theoretical findings.

Keywords. fractional differential equations, overlapping domain decompositions, preconditioners

1 Introduction

Space fractional partial differential equations have been widely used to describe the super-diffusion processes in the natural world (see [22]). Let Ω denote a polyhedral domain in $\mathbb{R}^d$, $M(z)$ denote a probability density function on S^{d-1}, where $S^{d-1} = \{z \in \mathbb{R}^d; ||z||_2 = 1\}$, and $|| \cdot ||_2$ denote the standard Euclidean norm. In this paper, we consider the following multi-dimensional SFPDE([18])

$$-\int_{S^{d-1}} D_z^{2\alpha} u(x)\, M(z) dz + cu(x) = f(x), \quad x \in \Omega, \tag{1.1}$$

where $1/2 < \alpha < 1$, $c \geq 0$ and $D_z^{2\alpha}$, which will be given later, denotes the directional derivative of order 2α in the direction z. We assume M is symmetric about origin, i.e., $M(z) = M(z')$ if $z, z' \in S^{d-1}$ satisfy $z + z' = 0$, which means that the above SFPDE is symmetric.

Actually, the equation (1.1) is an appropriate extension from one dimensional problem

$$-(p\ {}_{-\infty}D_x^{2\alpha} + q\ {}_xD_\infty^{2\alpha})u + cu = f, \tag{1.2}$$

and its corresponding developing equation can be used to describe a general super-diffusion process (see [18] for details), where ${}_{-\infty}D_x^{2\alpha}, {}_xD_\infty^{2\alpha}$ denote Riemann-Liouville fractional derivatives. One special case of (1.1) is

$$-\sum_{i=1}^{d}(p_i\ {}_{-\infty}D_{x_i}^{2\alpha} + q_i\ {}_{x_i}D_\infty^{2\alpha})u + cu = f \tag{1.3}$$

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†Department of Mathematics and Scientific Computing, Changsha University of Science and Technology, Changsha, 410076, China (jiangyingjun@csust.edu.cn).

‡School of Mathematical Sciences, Tongji University, Shanghai, China, and Institute of Computational Mathematics and Scientific/Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, P.O. Box 2719, Beijing, 100190, China(xxj@lsec.cc.ac.cn).