

# Accepted Manuscript

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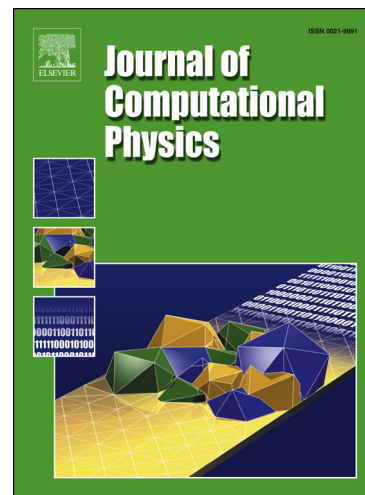
PII: S0021-9991(17)30647-2  
DOI: <http://dx.doi.org/10.1016/j.jcp.2017.08.066>  
Reference: YJCPH 7568

To appear in: *Journal of Computational Physics*

Received date: 9 May 2016  
Revised date: 24 April 2017  
Accepted date: 31 August 2017

Please cite this article in press as: Y. Jiang, X. Xu, Domain decomposition methods for space fractional partial differential equations, *J. Comput. Phys.* (2017), <http://dx.doi.org/10.1016/j.jcp.2017.08.066>

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# Domain Decomposition Methods for Space Fractional Partial Differential Equations\*

Yingjun Jiang<sup>†</sup> and Xuejun Xu<sup>‡</sup>

## Abstract

A two-level additive Schwarz preconditioner is proposed for solving the algebraic systems resulting from the finite element approximations of space fractional partial differential equations (SFPDEs). It is shown that the condition number of the preconditioned system is bounded by  $C(1 + H/\delta)$ , where  $H$  is the maximum diameter of subdomains and  $\delta$  is the overlap size among the subdomains. Numerical results are given to support our theoretical findings.

**Keywords.** fractional differential equations, overlapping domain decompositions, preconditioners

## 1 Introduction

Space fractional partial differential equations have been widely used to describe the super-diffusion processes in the natural world (see [22]). Let  $\Omega$  denote a polyhedral domain in  $\mathbb{R}^d$ ,  $M(z)$  denote a probability density function on  $S^{d-1}$ , where  $S^{d-1} = \{z \in \mathbb{R}^d; \|z\|_2 = 1\}$ , and  $\|\cdot\|_2$  denote the standard Euclidean norm. In this paper, we consider the following multi-dimensional SFPDE ([18])

$$-\int_{S^{d-1}} D_z^{2\alpha} u(x) M(z) dz + cu(x) = f(x), \quad x \in \Omega, \quad (1.1)$$

where  $1/2 < \alpha < 1$ ,  $c \geq 0$  and  $D_z^{2\alpha}$ , which will be given later, denotes the directional derivative of order  $2\alpha$  in the direction  $z$ . We assume  $M$  is symmetric about origin, i.e.,  $M(z) = M(z')$  if  $z, z' \in S^{d-1}$  satisfy  $z + z' = 0$ , which means that the above SFPDE is symmetric.

Actually, the equation (1.1) is an appropriate extension from one dimensional problem

$$-(p {}_{-\infty}D_x^{2\alpha} + q {}_xD_{\infty}^{2\alpha})u + cu = f, \quad (1.2)$$

and its corresponding developing equation can be used to describe a general super-diffusion process (see [18] for details), where  ${}_{-\infty}D_x^{2\alpha}, {}_xD_{\infty}^{2\alpha}$  denote Riemann-Liouville fractional derivatives. One special case of (1.1) is

$$-\sum_{i=1}^d (p_i {}_{-\infty}D_{x_i}^{2\alpha} + q_i {}_{x_i}D_{\infty}^{2\alpha})u + cu = f \quad (1.3)$$

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\*The work of the first author is supported by National Natural Science Foundation of China (No. 11571053), Hunan Provincial Natural Science Foundation of China (No. 13JJ1020). The work of second author is supported by National Natural Science Foundation of China (No. 11171335, 11225107).

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