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Multi-domain boundary element method for axi-symmetric layered linear acoustic systems

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A B S T R A C T

Homogeneous porous materials like rock wool or synthetic foam are the main tool for acoustic absorption. The conventional absorbing structure for sound-proofing consists of one or multiple absorbers placed in front of a rigid wall, with or without air-gaps in between. Various models exist to describe these so called multi-layered acoustic systems mathematically for incoming plane waves. However, there is no efficient method to calculate the sound field in a half space above a multi layered acoustic system for an incoming spherical wave. In this work, an axi-symmetric multi-domain boundary element method (BEM) for absorbing multi layered acoustic systems and incoming spherical waves is introduced. In the proposed BEM formulation, a complex wave number is used to model absorbing materials as a fluid and a coordinate transformation is introduced which simplifies singular integrals of the conventional BEM to non-singular radial and angular integrals. The radial and angular part are integrated analytically and numerically, respectively. The output of the method can be interpreted as a numerical half space Green's function for grounds consisting of layered materials.

1. Introduction

Homogeneous porous materials, i.e., acoustic absorbers, are the main tool for soundproofing, e.g., materials like rock wool or synthetic foam. For specific applications, e.g., room acoustics, different layers of absorbing materials, e.g., a fleece and a synthetic foam in front of a layer of air and a rigid wall, are combined to optimize the sound absorption, e.g., to extend the absorption to low frequencies the depth of the air gap can be increased or to improve absorption properties at high frequencies a sandwich system of multiple absorbing materials can be used to optimize the flow resistivity of the overall system.

Acoustic absorbers can be characterized by material parameters and acoustic parameters. The three main material parameters are the specific flow resistivity, the tortuosity and the porosity which can be used to calculate the acoustic parameters [1]. The acoustic property of an absorber is usually described either by the absolute-valued absorption coefficient or by the complex-valued wall impedance. The former parameter describes an energy ratio of an incoming and reflected

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wave, is usually measured in reverberation rooms, and is used in ray-tracing simulations [2], e.g., room acoustics simulations, where the absorption coefficient at boundary surfaces results in a simple amplitude reduction of reflected rays. The latter parameter is the complex ratio of an incoming and reflected *plane wave* where the absorbing material is assumed to be of infinite depth and is usually measured in impedance tubes. Note that also absorbers of finite depth are often reduced to an equivalent surface impedance, i.e., the normal impedance or normal admittance, and that the surface impedance is not a material parameter. The normal admittance is often used in numerical simulation methods, e.g. the boundary element method (BEM, [3]). However, the normal impedance is not an acoustic parameter of the absorbing material.

Acoustic absorbers are either locally or non-locally reacting [4]. In locally reacting materials, the waves propagate normal to the material surface only and, thus, the normal admittance of the boundary surface is sufficient to describe the acoustic properties of such an absorber. In non-locally reacting absorbers, the propagation of waves is not restricted to the normal direction of the material surface and, thus, the admittance is not constant at the material surface, but changes with the shape of the incident wavefront, and cannot be described by a normal admittance [5]. Since the simplification of acoustic absorbers to locally reacting surfaces and their approximation by surface normal admittances can result in simulation errors, e.g., in room acoustic simulations [6], advanced models have to be developed to calculate the reflection of spherical waves from non-locally reacting absorbers.

In the field of computational acoustics, absorbers are modeled in different ways depending on the modeling depth in the simulation method and a large number of works have been published on the mathematical modeling of layered absorbing materials [7–9]. For simple geometrical problems analytical formulations exist, e.g., for scattering of plane waves with normal incident direction from acoustic layered systems [10,11], for a point source above an impedance ground [12,13], or for a point source above an absorbing layer [14,15] where the absorber was modeled as fluid allowing the consideration of multiple reflections. Also the finite element method (FEM, [16]) can be used for small geometrical problems, e.g., to simulate the sound reflection from a layered system in a periodic unit cell [11]. For modeling as fluid, in the simulation a complex wave number is calculated from the absorber material parameters resulting in inherent sound damping in the fluid domain and sound wave reflections at the change of medium on the absorber boundaries. Since, in reality, absorbers are not infinitely thin and cause of the depth of an absorber, e.g., a 5 cm thick synthetic foam on walls of a listening room, multiple reflections occur [14]. Parts of the wave are reflected at the surface of the absorber and parts are reflected from the rigid surface behind the absorber.

In the present work, a boundary element method (BEM) to calculate the sound field of a spherical wave and its reflection above a system consisting of arbitrarily layered isotropic materials is derived. A BEM formulation was chosen, because the BEM has got several benefits outperforming the FEM, i.e., a trivial handling of point sources and of directive sources (spherical harmonic decomposition), a trivial handling of infinite domains, 1D instead of 2D integrals, an arbitrary number and position of evaluation points, and a simple truncation of the layer domains by truncation of the boundary integrals. Each layer, i.e., a subdomain in the BEM, has an infinite size in the x-y-plane and an arbitrary thickness in the z-direction. The thickness of a layer can even be infinite, e.g., half spaces of air above a layered system, because of the inherent compliance of the BEM to the Sommerfeld radiation condition. The absorbing material in an absorber subdomain is modeled as fluid, i.e., by a complex wave number [17,18].

For the proposed method, the general problems of the BEM had to be addressed, i.e., calculation of singular integrals and high computational effort. First of all, the BEM formulation itself reduced the discretization of the numerical problem to the subdomain interfaces, where the interface and boundary conditions are defined. The computational effort was further reduced by taking advantage of the axi-symmetry of the described numerical problem, i.e., at a subdomain interface the sound field of the spherical wave is constant on orbits around the x–y-position of the sound source. Note that such a symmetry had also been used in the so called ‘Fourier boundary element method’ [19], where the boundary integrals were separated into an angular and a radial part, i.e., the angular part was represented via a Fourier series and the radial part was integrated according to the boundary geometry. In our method, the axi-symmetry resulted in a discretization of the subdomain interfaces by an infinite set of ring elements around the sound source position. To handle the singular BEM integrals, a coordinate transformation was used which got rid of the singularities in the integrals and additionally allowed an analytical integration of the radial part of the boundary integrals. The remaining non-singular integrals over the angular part were then solved by conventional Gaussian quadrature.

The presented method not only enables the calculation of axi-symmetric sound fields above an admittance plane, e.g., for an incident spherical wave, but also the calculation of the sound propagation inside the layered absorbing system and, thus, multiple reflections to make the simplification to an admittance plane unnecessary in the first place. The output of the method can be interpreted as a numerical half space Green’s function for ground’s consisting of layered materials.

1.1. Boundary element method

The deduction of the boundary element method can be performed in the following steps. The starting point is the Helmholtz equation:

$$\triangle p(\vec{r}) + k^2 p(\vec{r}) = 0, \tag{1}$$

with $p(\vec{r})$ the sound pressure at position $\vec{r}$, $k = \frac{\omega}{c}$ the wavenumber, and ω the circular frequency. The Helmholtz equation can be solved if the Green’s function $G(\vec{r}, \vec{r}')$, fulfilling the definition