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On the use of kinetic energy preserving DG-schemes for large eddy simulation

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A B S T R A C T

Recently, element based high order methods such as Discontinuous Galerkin (DG) methods and the closely related flux reconstruction (FR) schemes have become popular for compressible large eddy simulation (LES). Element based high order methods with Riemann solver based interface numerical flux functions offer an interesting dispersion dissipation behavior for multi-scale problems: dispersion errors are very low for a broad range of scales, while dissipation errors are very low for well resolved scales and are very high for scales close to the Nyquist cutoff. In some sense, the inherent numerical dissipation caused by the interface Riemann solver acts as a filter of high frequency solution components. This observation motivates the trend that element based high order methods with Riemann solvers are used without an explicit LES model added. Only the high frequency type inherent dissipation caused by the Riemann solver at the element interfaces is used to account for the missing sub-grid scale dissipation. Due to under-resolution of vortical dominated structures typical for LES type setups, element based high order methods suffer from stability issues caused by aliasing errors of the non-linear flux terms. A very common strategy to fight these aliasing issues (and instabilities) is so-called polynomial de-aliasing, where interpolation is exchanged with projection based on an increased number of quadrature points. In this paper, we start with this common no-model or implicit LES (iLES) DG approach with polynomial de-aliasing and Riemann solver dissipation and review its capabilities and limitations. We find that the strategy gives excellent results, but only when the resolution is such, that about 40% of the dissipation is resolved. For more realistic, coarser resolutions used in classical LES e.g. of industrial applications, the iLES DG strategy becomes quite inaccurate. We show that there is no obvious fix to this strategy, as adding for instance a sub-grid-scale models on top doesn't change much or in worst case decreases the fidelity even more. Finally, the core of this work is a novel LES strategy based on split form DG methods that are kinetic energy preserving. The scheme offers excellent stability with full control over the amount and shape of the added artificial dissipation. This premise is the main idea of the work and we will assess the LES capabilities of the novel split form DG approach when applied to shock-free, moderate Mach number turbulence. We will demonstrate that the novel DG LES strategy offers similar accuracy as the iLES methodology for well resolved cases, but strongly increases fidelity in case of more realistic coarse resolutions.

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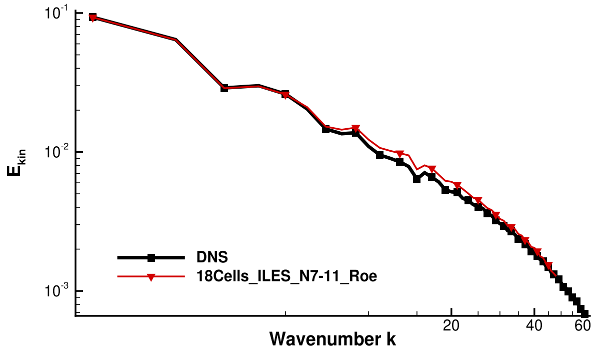
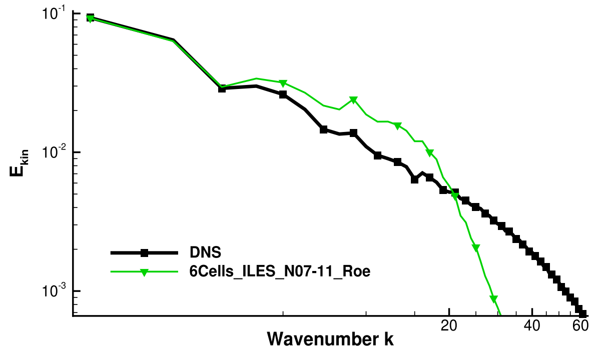


(a)



(b)

Fig. 1. Decaying homogeneous isotropic turbulence DNS/LES kinetic energy spectra: a) iLES 144^3 DOF, b) iLES 48^3 DOF. Both approximations use Roe's approximate Riemann solver at the element interfaces.

1. Introduction

Using high order Discontinuous Galerkin (DG) methods, and closely related methods such as flux reconstruction (FR) schemes, implicit large eddy simulation (iLES) approaches were recently applied to successfully simulate flows at moderate Reynolds numbers e.g. [41,7,3,4,14,42,33]. The approach used in these references does not add an explicit sub-grid scale model, such as e.g. eddy viscosity, but instead relies on the dissipation behavior of the schemes due to Riemann solvers at element interfaces. High order discretization, i.e. methods with polynomial degree $N \geq 3$, are usually used to obtain good dispersion behavior which is necessary to accurately capture the interaction of different spatial scales. Within the references above, it is shown that for very high order polynomial degrees, the iLES DG discretization are prone to aliasing issues due to the non-linearity of the flux functions. These aliasing issues can even cause instabilities and cause the simulation to crash. Within the references above, stabilization in case of very high order polynomial degrees is done by applying so-called polynomial de-aliasing. There are several ways of implementing and interpreting polynomial de-aliasing. The goal however is always the same: account for the non-linearity of the flux function by either using a polynomial projection or by increasing the number of quadrature points for the approximation of the integrals. In [25], Kirby and Karniadakis demonstrated that cutting off 1/3 of the highest modes in every time step for the incompressible Navier–Stokes equations (quadratic flux function) gives a stable approximation. They extended this for compressible flow, where they interpreted the non-linearity of the compressible Navier–Stokes fluxes as cubic and found that cutting off 1/2 of the highest modes gives stability in all cases they tested. The non-linearity in the compressible case is however rational and thus there are also cases where this strategy fails, see e.g. [33,19]. The issue of aliasing is mitigated somewhat by the use of Riemann solvers at element interfaces. For element based high order methods, the Riemann solver at the element interfaces provides numerical dissipation that acts in a high frequency filter-like way: it is almost zero for well resolved scales but gets very high for scales close to the Nyquist limit. This numerical dissipation behavior paired with the good dispersion properties is the motivation to use element based high order methods such as the DG scheme with interface Riemann solvers for under-resolved turbulence simulations, often termed implicit LES (iLES), as no explicit sub-grid scale dissipation model is added.

In the following, we will use the iLES DG approach to simulate the decay of homogeneous isotropic turbulence (DHIT). The test case is a homogeneous isotropic turbulence freely decaying for about one large eddy turn over time ($T = \bar{v}(t_{start})/L_{int} \approx 1.3$). The Reynolds number based on the Taylor micro-scale decays from $Re_\lambda \approx 162$ to $Re_\lambda \approx 97$. The LES initial state is obtained from a filtered DNS field. The DNS is simulated as in [43], with a pseudo-spectral code on 512^3 DOF using the 2/3 rule for de-aliasing. Similar test cases for LES were used for example in [23,21,14]. Fig. 1(a) shows the resulting kinetic energy (KE) spectra for a relatively high resolved LES setting, 18 cells per direction and polynomial degree $N = 7$, i.e. 144 DOF per direction in comparison with the DNS result. For the Riemann solver, we choose Roe's approximate numerical flux function. We note that in this case polynomial de-aliasing is applied according to the 3/2 rule (total of 12^3 quadrature points per element), as the flow behaves almost incompressible, see e.g. [16].

With this particular iLES DG setup about 62.5% of the total dissipation at t_0 can be resolved if we consider the theoretical Nyquist wavenumber $k_{Ny} = 144/2 = 72$ for this discretization with 144^3 DOF. Assuming a more realistic (but still optimistic) approximation capability of the piece-wise polynomial ansatz space of 3 points per wavenumber (PPW), then only about 43.2% of the KE dissipation is resolved. This setup is still somewhat well behaved: although under-resolved, the cut off wavenumber $k_c = 144/3 = 48$ falls within the beginning of the dissipation range, which is the reason that the results obtained with the DG iLES approach are in excellent agreement with the reference direct numerical simulation (DNS).

However, in the available literature, simulations of the DHIT test case usually aim at considerable lower resolutions. In the spectral community 32^3 points, e.g. [38], are commonly used for the assessments. Accounting for the difference between approximations with Fourier basis (2 PPW resolution limit) and polynomial basis (about 3 PPW resolution limit), we increase the DG resolution to 48^3 DOF, in particular we choose for the next setup 6^3 elements with a polynomial degree of $N = 7$.